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A Study of Aerodynamic Matrix Numerical Condition

By

Dean Bellinger and Tony Pototzky
MSC.Software Corporation NASA Langley Research Center

Abstract

In MSC.Nastran the aerodynamic matrix is calculated and manipulated to form generalized aerodynamic influence coefficients (AIC). The AIC matrix can be perceived as a complex stiffness matrix in the general equations of motion. The numerical condition of the aerodynamic matrix is not as fully evaluated by MSC.Nastran in the aeroelastic analysis solution sequences as it could be. A study is made to evaluate the numerical behavior of the aerodynamic matrix for both subsonic and supersonic conditions using matrix tools from Version 2001 of MSC.Nastran. The aerodynamic matrix determinant value and the singular value decomposition terms are calculated and summarized for a few sample wing planform configurations.

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Introduction

The aeroelastic capability of MSC.Nastran generates aerodynamic influence coefficient (AIC) matrices that can be thought of as complex aerodynamic stiffness matrices. The user's guide by Rodden and Johnson[1] describes the aeroelastic capability of MSC.Nastran. The AIC matrices are generated from aerodynamic matrices calculated by the subsonic Doublet-Lattice or the supersonic ZONA51 methods. The complex/unsymmetric AIC matrices do not allow the symmetric structural matrix-to-factor diagonal ratio to be used to measure the numerical condition. This study is performed to determine if the matrix determinant or the singular value decomposition (SVD) provides a means of measuring the numerical characteristics and condition of the aerodynamic matrices and determine the simulation suitability of the aerodynamic matrices. This latter item leads to the real purpose of this study: "Can the determinant or singular value decomposition identify when the AIC matrix becomes unusable for aeroelastic calculation?"

A series of typical planform configurations are investigated in an attempt to answer the above question and to determine the numerical condition/characteristic of the aerodynamic matrices. Both subsonic and supersonic conditions are investigated and summarized in this study. The planform parameters, e.g., sweep and taper, are also part of this study. A delta wing is investigated as the final planform configuration. Data for the determinant and SVD value variation with reduced frequency are presented to determine if they can be used to answer the above question.

Model Description

Aerodynamic surface planforms from Touvila and McCarty (NACA RM L55E11) were used in the study to investigate the determinant (Det) and singular value decomposition ratio (SVDR). The planforms consist of three basic configurations. The first planform configuration uses four models having constant chord and four sweep angles of 15, 30, 45 and 60 degrees. The second planform configuration uses two delta wings of 45 and 60 degrees of sweep. The third planform configuration combines sweep and taper, only two of the five configurations described by Touvila and McCarty were used. Of the tapered models chosen for this study, one has a taper ratio, λ, of 0.2 and no sweep at the quarter-chord and the second has a taper ratio of 0.4 and 45 degrees of sweep. All models were examined at one subsonic and one supersonic Mach number. The models have a plane of symmetry as shown by the wing planforms in Figures 1 and 2.

The two planform figures list the Mach number, sweep and taper variations made to conduct this study. The 15^o sweep model of Figure 1 is a well-publicized model used by Rodden and Johnson[1] to perform static, flutter and response aeroelastic analyses. The aerodynamic model representation used in Rodden and Johnson[1] has four chordwise (streamwise) and six spanwise elements. The aerodynamic element mesh for the constant chord aerodynamic model is one of six aerodynamic models used in this study. The other five aerodynamic models increment the number of chordwise elements by four elements while maintaining the same element aspect ratio. The highest value of reduced frequency, k, of Rodden and Johnson[1] for the flutter and

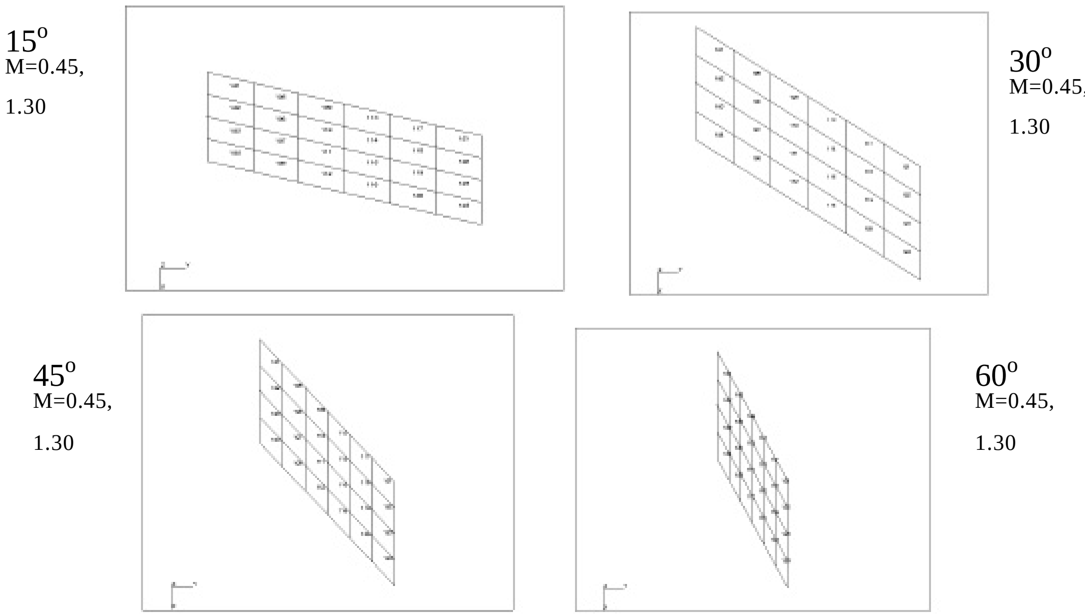

Figure 1. – Constant Chord Example Planforms with Four Sweep Angles and Two Mach Numbers.

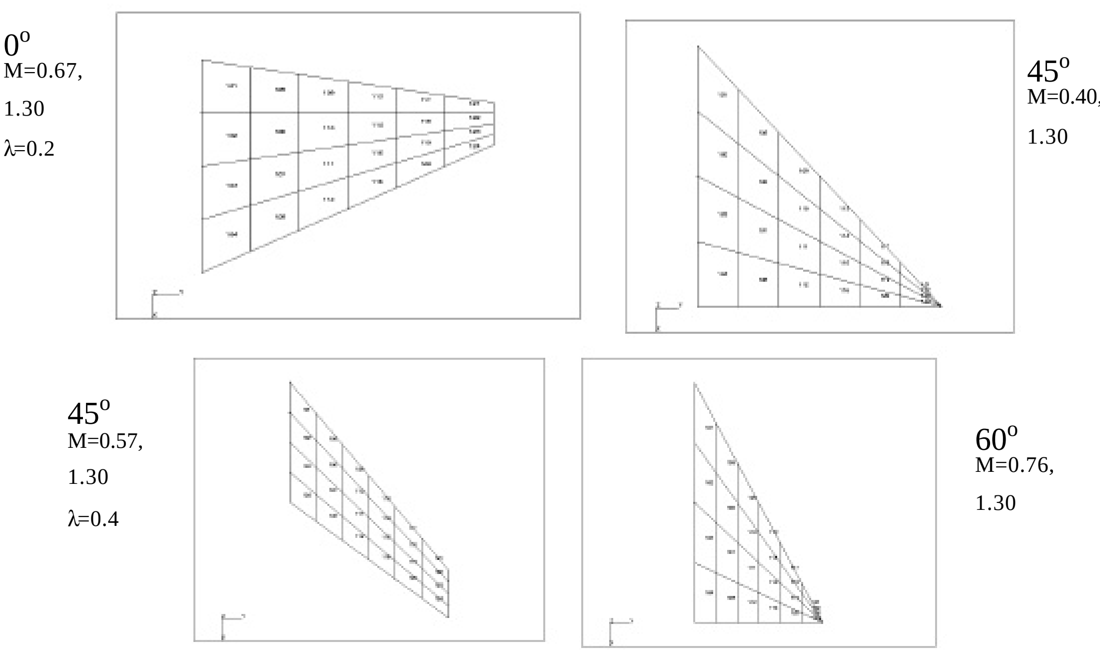

Figure 2. – Taper and Delta Wing Example Planforms at Two Sweep Angles, Two Mach Numbers and Two Taper Ratos.

response analyses are 0.20. This value of reduced frequency is well within the acceptable range according to Rodden and Johnson's[1] aerodynamic modeling guidelines in Section 3.1. This modeling rule has since been revised and reported by Rodden at the June 1999 Aerospace Flutter and Dynamics Council Meeting. The guideline of Rodden an Johnson[1] yields the following equation:

$$N_{cbox} = (12/\pi)k \qquad (1)$$

The revised guideline yields the following relationship of number of chordwise boxes to the reduced frequency.

$$N_{cbox} = (48/\pi)k \qquad (2)$$

Aerodynamic Matrix Processing

Two approaches for evaluating the numerical behavior of the aerodynamic matrix are employed in this study. The determinant of the matrix is one way of learning about the aerodynamic matrix. Numerical difficulties are likely to be encountered during the decomposition of a matrix when the determinant tends to zero. However, this presents a problem because the more detail of the aerodynamic mesh, the determinant may grow very large or very small. In the cases investigated in this study, the determinant grew smaller with increasing mesh density. So a better method of evaluating the numerical condition of the aerodynamic matrix was sought for this study. One approach is available from th CEAD module of MSC.Nastran for Version 70.6 and later version. The approach is the Singular Value Decomposition of a matrix. Numerous references describe this approach and Golub and Van Loan[3] provide valuable insight with regard to the determination of the numerical condition of a matrix. The determinant is not a viable measure of numerical condition; however, its behavior seems to indicate some puzzling evidence as shown later in this report. The SVD shows some valuable information about the ZONA51 aerodynamic matrix. Figures 3 through 6 display the actual maximum and minimum value

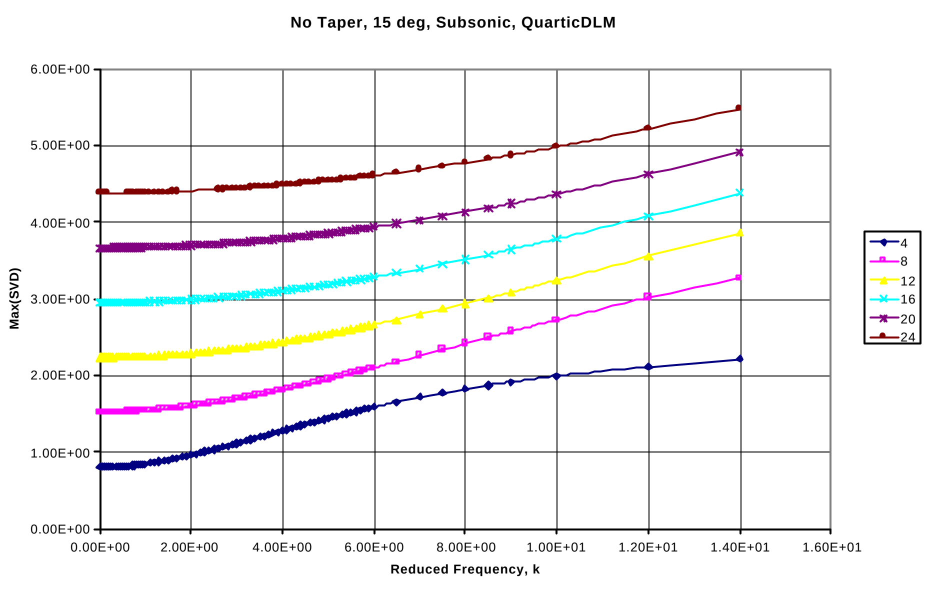

Figure 3. – Maximum Singular Value Decompostion Values for the Quartic-DLM Aerodvnamic Matrix.

output by the SVD value method for the aerodynamic matrices of the quartic-DLM and the ZONA51 methods. The maximum and minimum values were used to create a ratio of these two values for more convenient evaluation.

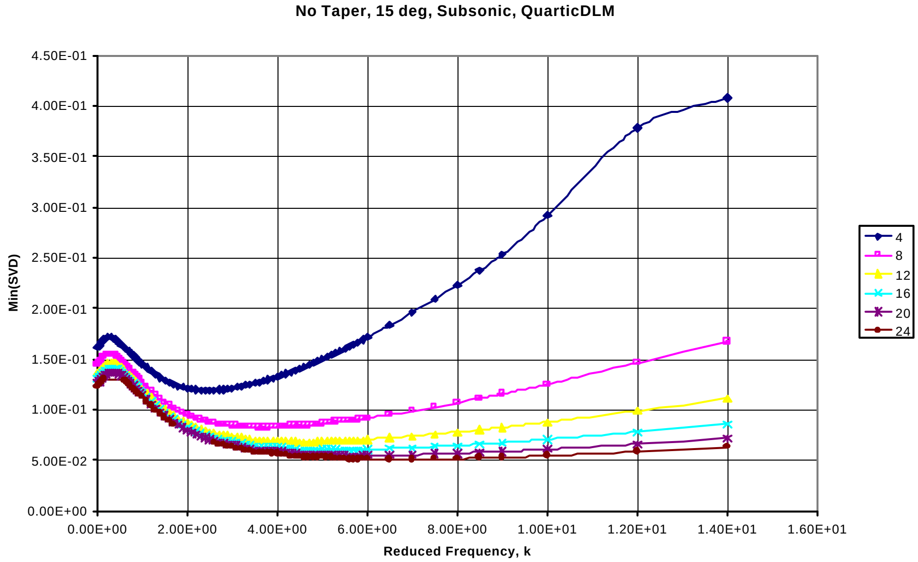

Figure 4. – Minimum Singular Value Decompostion Values for the Quartic-DLM Aerodynamic Matrix.

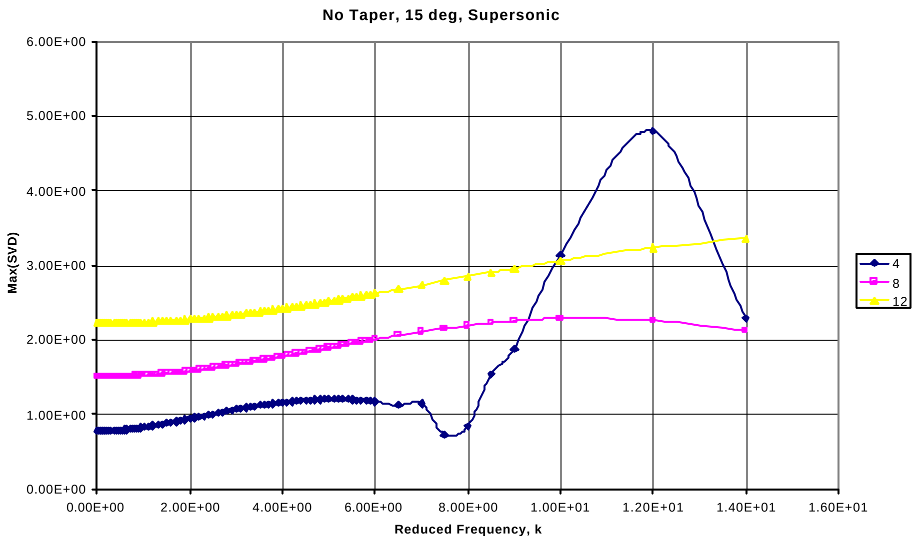

Figure 5. – Maximum Singular Value Decompostion Values for the ZONA51 Aerodynamic Matrix.

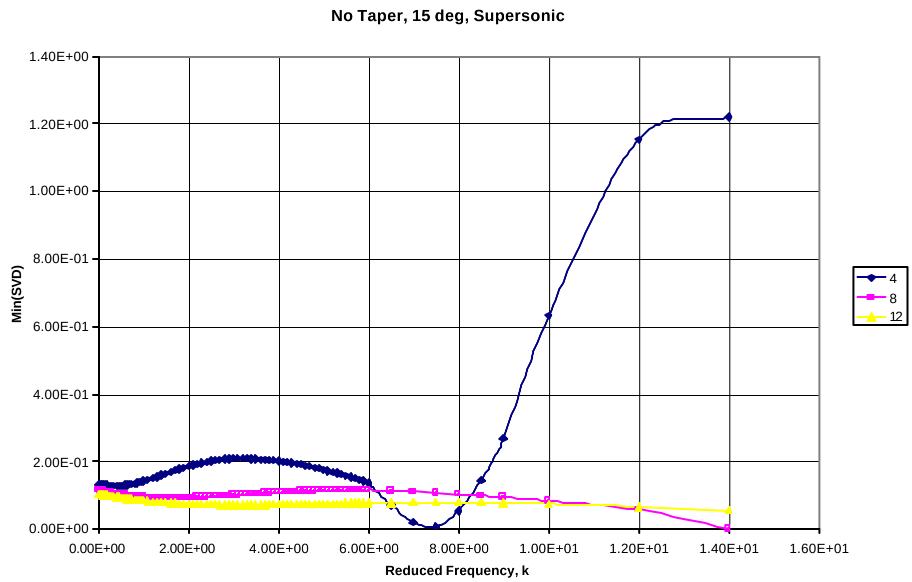

Figure 6. – Minimum Singular Value Decompostion Values for the ZONA51 Aerodynamic Matrix.

A method to process and present the Det and SVD data is required to fulill this study. A DMAP alter is created for SOL 145, aeroelastic flutter analysis, to compute the Det and SVD values for the configurations shown in Figures 1 and 2. The Det and SVD values are normalized with respect to the Det and SVD values at a reduced frequency, k, of 0.001 as given in Equations 3 and 4.

$$\mathrm{Nrm(Det)} = \frac{\mathrm{Det(m,k)}}{\mathrm{Det(m,0.001)}} \tag{3}$$

$$\mathrm{Nrm(SVDR)} = \frac{\mathrm{SVD(m,k)}}{\mathrm{SVD(m,0.001)}} \tag{4}$$

The DMAP alter and typical input data file are presented in Listings 1 and 2 at the end of this report. The DMAP alter calculates and formats the Det and SVD values to facilitate importing into Microsoft Excel. Excel provides a general method of xy-plotting of results presentation and comprehension. The models were setup to output the Det and SVD values for all of the model configurations shown in Figures 1 and 2 above. The MKAERO1 entry used with each planform contained 84 k values and one Mach number to describe the Det and SVD value behavior versus k. Normally, this quantity of reduced frequencies is not required to perform an aeroelastic analysis. A large number of k values is used to study the Nrm(Det) and Nrm(SVDR) variation with reduced frequency and ensure that some numerical anomaly is not overlooked as will be seen in some of the SVDR data. Results for each planform are determined for one subsonic and one supersonic Mach number.

Results

Table 1 summarizes the runs made to obtain the Det and SVD values. The 15 degree swept wing model has the most indepth analysis of the suite of models. Note that this model is run with quadratic and quartic DLM for the six chordwise aerodynamic element variation. This model shows some interesting behavior of the Det values in the subsonic case. The supersonic case is run with the coarser mesh sets of aerodynamic elements. The tapered and delta wing configurations are run with the least number of chordwise aerodynamic elements except for Quartic DLM aerodynamics.

For discussion purposes, only the results of the 15-degree swept wing model will be presented. The Det and SVDR values are given in Figures 7 – 10 for subsonic case using quadratic and quartic DLM aerodynamics. The Det values in Figure 7 for the quadratic-DLM generated AIC's tend to zero indiciating a singular matrix at higher reduced frequencies. However, the SVDR values in Figure 8 for the same matrices indicate well- conditioned matrices making them

Table 1. – Summary of Model Runs

		Number of Chordwise Boxes					
Model Description	Sweep	4	8	12	16	20	24
Constant Chord, i.e., no taper	15 30 45 60	1 x 2 1 x 2 1 x 2 1 x 2	1 x 2 1 x 2 1 x 2 1 x 2	1 x 2 1 x 2 1 x 2 1 x 2	1 x 2	1 x	1 x
Tapered	0 45	1 2 1 2	1 2 1 2				
Delta	45 60	1 x 2 1 x 2	1 x 2 1 x 2	x x			

1- Quadratic DLM, x – Quartic DLM, 2 – ZONA51

inconsistent with the Det. Nevertheless, for the same aero model, the Det values in Figure 9 from the quartic-DLM generated matrices indicate an opposite behavior of increasing with increasing frequency instead of going to zero. Again, the SVDR values in Figure 10 show a similar trend as Figure 8, especially for the greater aero mesh denstiy. Figures 11 and 12 show the model using supersonic aerodynamics from the ZONA51 method. Interestingly, both Det and SVDR values of Figures 11 and 12 are consistent in showing singular matrices at approximately reduced frequencies of 7.3 and 14 for the 4 and 8 box cases, respectively. For comparison with the Det and SVDR value variation over reduced frequency, a generalized aerodynamic influence coefficient is given in Appendix A. The values shown in Appendix A are from the constant chord-15^{o} swept wing model at subsonic and supersonic

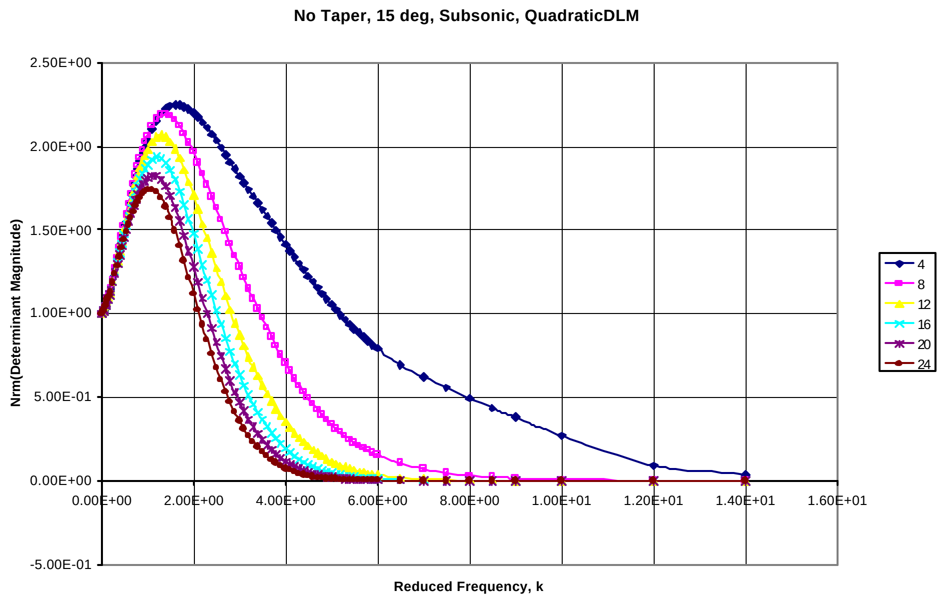

Figure 7. – Normalized Determinant of Quadratic DLM Aerodynamic matrix as a Funciton of Reduced Frequency.

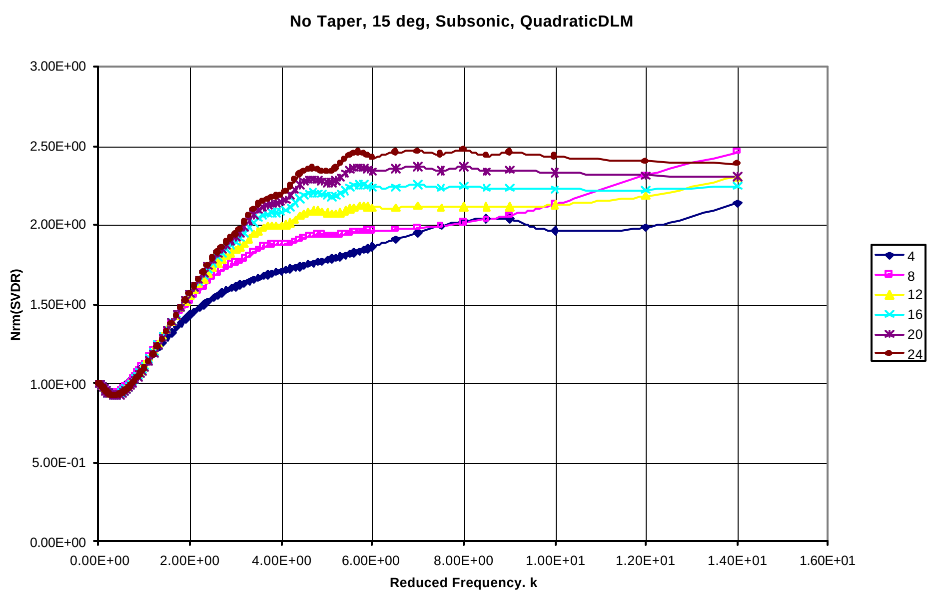

Figure 8. – Normalized Singular Value Decomposition Ratio Variation with Reduced Frequency for Quadratic DLM Aerodynamics.

speeds for a rigid body pitch mode. The pitch mode generalized aerodynamic influence coefficients for the 4 and 8 box supersonic cases in Figures A3 and A4 show the erratic tendencies above the reduced frequency of 7 and near the reduced frequency of 14. These tendencies are consistent with the results shown in Figures 11 and 12.

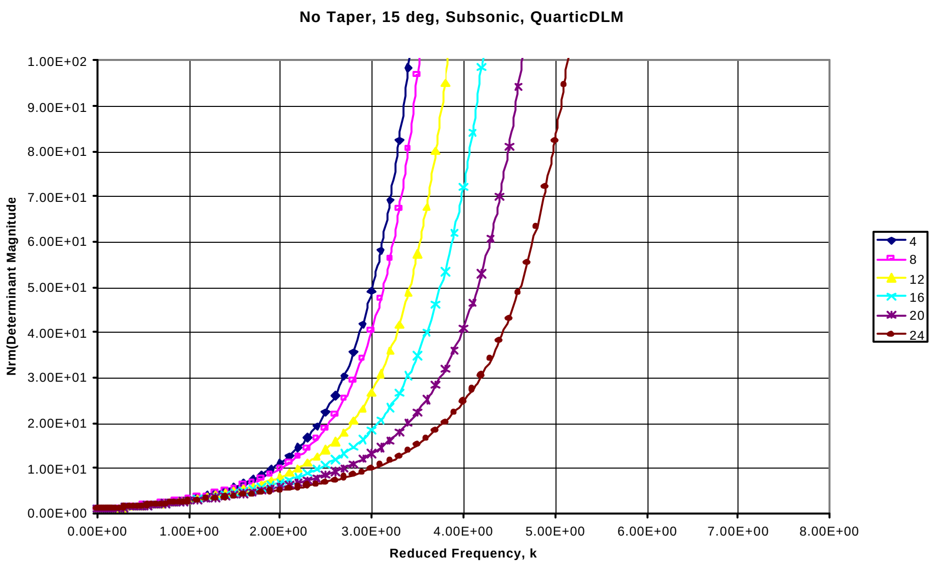

Figure 9. – Normalized Determinant Variation with Reduced Frequency for Quartic DLM Aerodynamics.

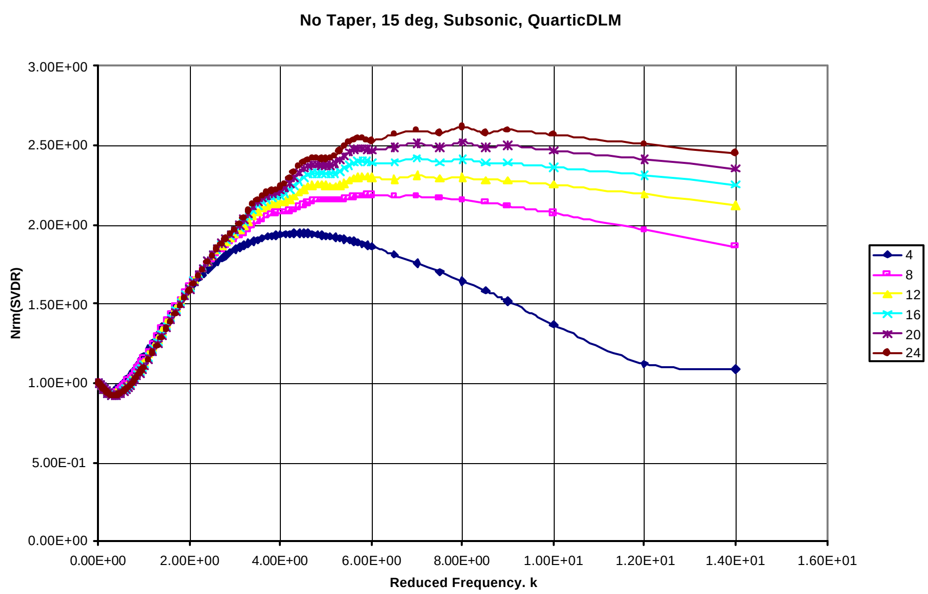

Figure 10. – Normalized Singular Value Decomposition Ratio Variation with Reduced Frequency with Quartic DLM Aerodynamics.

Admittedly, the range of reduced frequencies is purposely specified over an extreme range as seen in the above figures. While it is not necessary or prudent to perform a flutter or aeroelastic response analysis. The internal interpolation capabilities over the MKAERO1 reduced frequency range by the FA1 module means that a more frugal number of reduced frequencies can be use to determine the effect of any intermediate reduced frequency with any of the available flutter

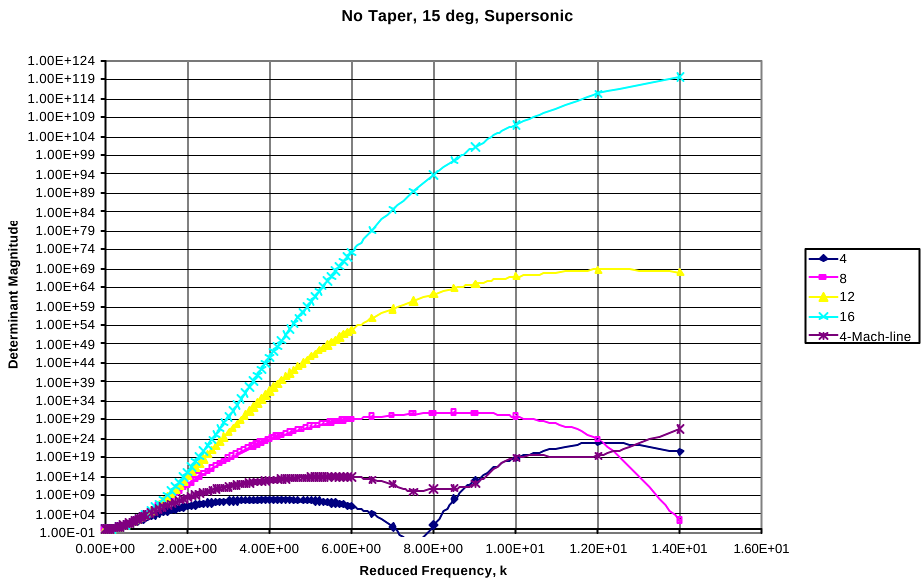

Figure 11. – Normalized Determinant Variation with Reduced Frequency for Supersonic ZONA51 Aerodynamics.

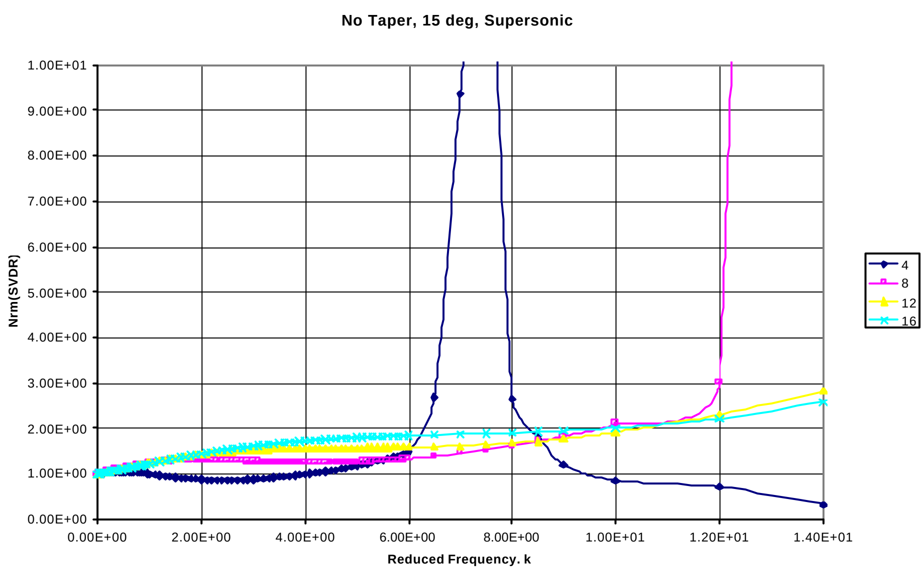

Figure 12. – Normalized Singular Value Decomposition Ratio Variation for Reduced Frequency with ZONA51 Aerodynamics.

methods of the program. A large number of reduced frequencies are used in this study to ensure continuity of the Det and SVDR values. For practical reasons the reduced frequencies above 6.0 are more widely spread than below the 6.0 value.

Conclusions

More varied aerodynamic configurations need further study with additional mesh density to better understand the numerical characteristics.

SVD demonstrates the numerical condition of the aerodynamic matrices. DLM behavior appears to be without numerical difficulty. ZONA51 shows poor numerical condition for coarsest meshes above reduced frequencies of 6.0.

When using the Quartic-DLM method, the determinant value is relatively consistent in its behavior. Quadratic-DLM and ZONA51 methods are more erratic at the higher reduced frequencies and are planform dependent.

The SVDR values do not demonstrate any excessive values and the DLM matrices appear to be numerically well conditioned. The onset of the large amplitude variation of the determinant magnitude or the SVDR value only occurs after the guidelines established in Equation 2 are exceeded. If one applies the guideline then any numerical problems with the aerodynamic matrix is avoided. In answer to the question raised at the beginning of the report, in general, the determinant develops high values for the quartic-DLM at the higher reduced frequencies, but this is exhibited after violation of the guideline of Equation 2.

Acknowledgement

With the assistance of Tom Kowalski of MSC.Software Corporation, the singular value decomposition matrix was implemented as part of this study.

References

1. Rodden, W. P., and Johnson, E. H., “MSC/NASTRAN Aeroelastic Analysis User’s Guide,” Version 68, October 1994.
2. Touvila, W. J., and McCarty, J. L., “Experimental Flutter Results for Cantilever Wing Models at Mach Numbers Up to 3.0,” NACA RM L55E11, 1955.
3. Golub, H. G., and Van Loan, C. F., Matrix Computations, The Johns Hopkins University Press, Baltimore, Maryland.

Listing 1. – DMAP Alter to Output Det and SVD Ratios

```
$
$ A DMAP alter to output Singular Value Decomposition
$ maximum/minimum matrix ratio, SVDR, values and the
$ Determinant of the AJJT matrix calculated from DLM
$ or ZONA51 aerodynamic methods.
$
$ by Dean Bellinger, October, 1999.
$
$ If CHKAJJT is set to YES with PARAM,CHKAJJT,YES in the
$ bulkdata section of input then the DECOMP module is called
$ so that it deselects the SPARSE decomposition method.  This
$ write the DET and POWER output parameters from the DECOMP
$ module to a USERFILE on Fortran unit 25.  Additionally, a
$ parameter CEIGAJJT can be set to YES to calculate the
$ Singular Value Decomposition matrix from the CEAD module
$ so that the SVDR value can be calculated and output.  The
$ SVDR values is also written to the userfile on Fortran unit 25.
$ If the AJJT matrix is large, the CPU costs can be large.
$ However, the SVDR is a good measure of the numerical condition
$ of the AJJT matrix.
$
$ User Input:
$
$ File Management Section -
$
$ assign userfile='15d-045-24b.f25' unit=25 form=formatted delete new
$
$ Bulk Data -
$
$ PARAM,CHKAJJT,YES  (default is NO)
$ PARAM,CEIGAJJT,YES  (default is NO)
$
COMPILE PFAERO SOUIN=MSCSOU NOLIST NOREF
$
ALTER 'DECOMP.*AJJT','DECOMP.*AJJT' $
TYPE     PARM,,CHAR8,Y,CHKAJJT='NO      ' $
      IF (CHKAJJT='YES') THEN $
        CALL    CHKAJJT  AJJT, CASEAA / LAJJT, UAJJT /
                         S,KBAR / S,MACHNO / S,KCNT $
      ELSE $
        DECOMP   AJJT/LAJJT,UAJJT, $
      ENDIF $ CHKAJJT
COMPILE CHKAJJT NOREF NOLIST
$
SUBDMAP CHKAJJT A,CASEAA/L,U/KBAR/MACHNO/KCNT $
$
TYPE     DB,DYNAMICS $
TYPE     PARM,,CHAR8,Y,CEIGAJJT='NO      ' $
TYPE     PARM,,I,N,NOGOOD,BAD,KCNT $
TYPE     PARM,,RS,N,KBAR,MACHNO $
$
$ FIND COMPLEX EIGENVALUES OF AJJT
$
IF (CEIGAJJT='YES     ') THEN $
$
$ compute the aero matrix Singular Value Decomposition values
$
  CEAD     A,,,,,,/,CLAMA,,,svals/
           S,N,NCEIGV//-1/'svd'//0 $
  MESSAGE  //'SINGULAR VALUE DECOMP. FOR AJJT MATRIX AT K OF:'/KBAR $
  OFP      CLAMA // $
$
diagonal   svals/svdiag/'column'/1.0 $ extract diagonal of svals
diagonal   svdiag/svdiagm1/'whole'/-1.0 $ reciprocal of each svdiag terms
matmod     svdiag,,,,,/svdmax,/7 $ find maximum of svals diagonal terms
matmod     svdiagm1,,,,,/svdminm1,/7 $ find maximum of 1/svals diag. terms
add        svdmax,svdminm1/svdrat///1 $ SVDR (should be 1x1 matrix)
matprn     svdrat // $
paraml     svdrat//'dmi'/1/1/s,n,svdrat $
```

```
$  MATPRN   CPHDX,LPHDX // $ Uncomment for Eigenvector output
$
  MATMOD   A,,,,,/ACC,/10 $
  TRNSP    ACC/ACCT $
  MPYAD    A,ACCT,/AACCT/////6 $
  MATMOD   AACCT,,,,,/RAACCT,IAACCT/34 $
  PARAML   RAACCT//'TRAILER'/1/S,N,NCRAACCT $
  MATGEN,   /RIDENT/1/NCRAACCT $
  READ     RAACCT,RIDENT,,,DYNAMICS,,CASEAA,,,,,,/LAMRAACC,
           VRAACCT,MRAACCT,OEIGS,,/'MODES'/S,N,NERAACCT/1 $
  OFP      LAMRAACC,OEIGS // $
$
  paraml   svdmax//'dmi'/1/1/s,n,svdmax $
  paraml   svdminm1//'dmi'/1/1/s,n,svdmin $
  svdmin = 1.0/svdmin $
ENDIF $ CEIGAJJT
$
$ DECOMPOSITION COMPUTE THE DETERMINANT
$
$ Setting of SYS209 to deactivate sparse method is required
$ to obtain the determinant of the Ajj matrix.
$
PUTSYS(0,209) $ DEACTIVATE SPARSE UNSYMMETRIC DECOMPOSITION
$
DECOMP   A/L,U,/-1//S,N,MINDIAG/S,N,DET/S,N,POWER/S,N,SING/
         S,N,NBRCHG/S,N,MAXRAT $
NOGOOD = 0-NBRCHG $
IF ( NOGOOD<0 OR SING<0 ) BAD=-1 $
IF (BAD = -1) THEN $
  MESSAGE  //'THE AJJT MATRIX IS PROBABLY SINGULAR.'/
             ' THIS MAY BE CAUSED BY PLACE A PANEL ON'/
             ' A PLANE OF SYMMETRY.' $
$  PRTPARM  // $ For debug output
  EXIT $
ELSE $
$
$ Set up output so it is easy to import into Microsoft Excel
$
  putsys(25,2) $ put following message in unit 25 file
$
$  MESSAGE  //'RED FREQ:'/KBAR/
$              '  DET MAN:'/DET/'  POW 10:'/POWER/
$              '  SVD RAT:'/svdrat/svdmax/svdmin $
$            2       3      4      5      6      7      8      9
  IF ( KCNT <= 2 ) MESSAGE   //'  MACH NO.  '/'    RED. FREQ.'/
                   '  RL(DET.MAN.)'/'IM(DET.MAN.)'/'     POWER'/
                   'SVD Ratio'/'  MAX SVD'/'    MIN SVD' $
  MESSAGE  //MACHNO/KBAR/DET/POWER/SVDRAT/SVDMAX/SVDMIN $
$
  putsys(6,2) $ reset to normal output unit 6
$
ENDIF $ BAD
PUTSYS(1,209) $ RESET SYSTEM CELL 209 TO DEFAULT
RETURN $
END $ CHKAJJT
```

Listing 2. – Sample Input Data File for 15° Swept Wing Constant Chord -Subsonic Model

```
nastran mesh
assign userfile='15d-045-24b.f25' unit=25 form=formatted delete new
ID MSC, chk-ajjt $ EDB - 27 Oct 1999
$$$$$$$$ FIFTEEN SWEEP                                 $$$$$$$$
$                                                              $
$        MODEL DESCRIPTION MODEL A OF NACA RM L55E11 $
$                                  15 DEGREE SWEPT WING        $
$                                  QUAD4 MODEL                 $
$                                                              $
$        SOLUTION         KE FLUTTER ANALYSIS METHOD$
$                                  USING DOUBLET LATTICE METHOD$
$                                  AERODYNAMICS                $
$                                                              $
$        RUN PRODUCES XY PLOTS OF THE V-G FLUTTER DATA$
$                 AND STRUCTURE PLOTS                          $
$                                                              $
$$$$$$$$                                               $$$$$$$$
TIME 10 $
diag 8,56
SOL 145 $ FLUTTER ANALYSIS
include 'ajjt-chka.v705'
CEND
TITLE = 15-DEG SWEPT WING (DLM AERODYNAMICS) 4 CHORDWISE BOXES     chk-ajjt
SUBT  = MACH 0.45 QUAD4 Plate model
LABEL = KE METHOD FLUTTER SOLUTION
  ECHO    = SORT
  SPC     = 1  $ WING ROOT FIXED
  METHOD  = 1  $ LANCZOS
  cmethod = 20 $ HESS
  FMETHOD = 30 $ KE-FLUTTER METHOD
  SET 1 = 1 THRU 124 $ PHYSICAL GRIDS
  DISP(PLOT) = 1 $
OUTPUT(PLOT)
  CSCALE 2.0
  PLOTTER NASTRAN
  SET 1  = AERO1,QUAD4
  SET 2  = QUAD4
  VIEW 34.,23.,0.
  PTITLE = STRUCTURAL ELEMENTS
  FIND SCALE, ORIGIN 1, SET 2
  PLOT MODAL 0 ORIGIN 1, SET 2
  PTITLE = STRUCTURAL AND AERODYNAMIC ELEMENTS
  FIND SET 1
  PLOT MODAL 0 ORIGIN 1, SET 1 SYMBOL 6 VECTOR R
$
  VIEW 0.,90.,0.
  FIND SCALE, ORIGIN 1 ,SET 1
  PLOT ORIGIN 1, SET 1, LABEL BOTH
$
  MAXIMUM DEFORMATION 1.-15
  PTITLE = STRUCTURAL ELEMENTS
  FIND SCALE, ORIGIN 1 ,SET 2
  CONTOUR ZDISP
  PLOT MODAL 0 CONTOUR OUTLINE ORIGIN 1, SET 2
OUTPUT (XYOUT)
  CSCALE 2.0
  PLOTTER NASTRAN
  CURVELINESYMBOL = -6
  YTTITLE = DAMPING  G
  YBTITLE = FREQUENCY  F  Hz
  XTITLE  = VELOCITY  V (in/sec)
  XTGRID LINES = YES
  XBGRID LINES = YES
  YTGRID LINES = YES
  YBGRID LINES = YES
  UPPER TICS  = -1
  TRIGHT TICS = -1
  BRIGHT TICS = -1
  XYPLOT VG / 1(G,F) 2(G,F) 3(G,F) 4(G,F) 5(G,F) 6(G,F)
```

```
BEGIN BULK
param,chkajjt,yes
param,ceigajjt,yes
param,post,0
eigc    20      hess    max                                     24
$***                                                                    ***$
$***    15 DEG SWEPT WING GRID POINTS                                   ***$
$***                                                                    ***$
EGRID   11              -1.0353 0.      0.
EGRID   12              .44517  5.5251  0.
EGRID   13              2.5157  5.5251  0.
EGRID   14              1.0353  0.      0.
GRIDG   1                       12      -11     -12     -13             +GG1
+GG1    -5      -14
LIST    5       .25     .5      .5      .5      .25
GRIDU   1                               1       THRU    78
$***                                                                    ***$
$***    15 DEG SWEPT WING COORDINATE SYSTEM AND ROOT CONSTRAINTS ***$
$***                                                                    ***$
CORD2R  1               .0      .0      .0      .0      .0      1.      +C1
+C1     .96593  -.25882 .0
$ comment out the next 3 lines for free model
SPC1    1       4       14      53
SPC1    1       12356   14      27      40      53
SPC1    1       16      1       THRU    78
SPC1    1       6       1       THRU    13
SPC1    1       6       15      THRU    26
SPC1    1       6       28      THRU    39
SPC1    1       6       41      THRU    52
SPC1    1       6       54      THRU    78
$ uncomment the next 6 lines for free model
$SPC1   1       126     99
$suport 99      345
$GRID   99
$RBE2   99      99      123456  14      27      40      53
$CONM2  90      99              1.+3
$       1.+4            1.+4                    1.+4
$***                                                                    ***$
$***    15 DEG SWEPT WING STRUCTURAL ELEMENTS                   ***$
$***                                                                    ***$
CGEN    QUAD4   1       1       1                       1       12      +LE
+LE     .000    .000    .312    .312
CGEN    QUAD4   1       1       1                       13      48
CGEN    QUAD4   1       1       1                       49      60      +TE
+TE     .312    .312    .000    .000
PSHELL  1       1       .041    1               1
$***                                                                    ***$
$***    15 DEG SWEPT WING MATERIAL PROPERTIES  ( ALUMINIUM )    ***$
$***                                                                    ***$
MAT1    1       10.4+6  3.9+6           2.61-4                          ALUMINUM
PARAM   COUPMASS1
$***                                                                    ***$
$***    15 DEG SWEPT WING AERODYNAMIC ELEMENT DESCRIPTION       ***$
$***                                                                    ***$
$AERO   ACSID   VELOCITY REFC   RHOREF  SYMXZ   SYMXY
AERO    0               2.0706  1.1092-7 1
$CAERO1 EID     PID     CP      NSPAN   NCHORD  LSPAN   LCHORD  IGID    +CONT
$CONT   X1      Y1      Z1      X12     X4      Y4      Z4      X43
CAERO1  101     1       1       6       4                       1       +CA101
+CA101  -1.0    -.26795 .0      2.0706  -1.     5.45205 0.0     2.0706
$MKAERO1 M1     M2      M3      M4      M5      M6      M7      M8      +CONT
$CONT    K1     K2      K3      K4      K5      K6      K7      K8
MKAERO1 .45                                                             +MK
+MK     .001    .025    .05     .075    0.1     .125    0.15    0.175
MKAERO1 .45                                                             +MKA
+MKA    0.2     0.25    0.3     0.35    0.4     0.45    0.5     0.55
MKAERO1 .45                                                             +MKA1
+MKA1   0.6     0.65    0.70    0.75    0.8     0.85    0.9     0.95
MKAERO1 .45                                                             +MKA2
+MKA2   1.0     1.1     1.2     1.3     1.4     1.5     1.6     1.7
MKAERO1 .45                                                             +MKA3
```

```
+MKA3    1.8      1.9      2.0      2.1      2.2      2.3      2.4      2.5
MKAERO1  .45                                                                +MKA4
+MKA4    2.6      2.7      2.8      2.9      3.0      3.1      3.2      3.3
MKAERO1  .45                                                                +MKB
+MKB     3.4      3.5      3.6      3.7      3.8      3.9      4.0      4.1
MKAERO1  .45                                                                +MKBB
+MKBB    4.2      4.3      4.4      4.5
MKAERO1  .45                                                                +MKB1
+MKB1    4.6      4.7      4.8      4.9      5.0      5.1      5.2      5.3
MKAERO1  .45                                                                +MKB2
+MKB2    5.4      5.5      5.6      5.7      5.8      5.9      6.0      6.5
MKAERO1  .45                                                                +MKC
+MKC     7.0      7.5      8.0      8.5      9.0      10.      12.      14.
$PAERO1  PID      B1       B2       B3       B4       B5       B6
PAERO1   1
$SET1    SID      G1       G2       G3       G4       G5       G6       --ETC.--+CONT
SET1     100      1        5        9        13       27       31       35       +S1
+S1      39       66       70       74       78
$SET1    100      1        THRU     26       27       THRU     78
$SET1    100      1        thru     13       14       thru     26       27
$        28       thru     39       40       thru     78
$param   opgtkg   0
$param   opgeom   0
$SPLINE1 EID      CAERO    BOX1     BOX2     SETG     DZ
SPLINE1  100      101      101      124      100      .0
$***                                                                   ***$
$***     15 DEG SWEPT WING EIGENVALUE AND FLUTTER CONTROL DATA     ***$
$***                                                                   ***$
PARAM    OPPHIPA  1
ASET1    3        1        THRU     13
ASET1    3        15       THRU     26
ASET1    3        28       THRU     39
ASET1    3        41       THRU     52
ASET1    3        54       THRU     65
ASET1    3        66       THRU     78
eigrl    1                          6
EIGR     10       MGIV                                6                         +ER
+ER      MAX
$FLFACT  SID      F1       F2       F3       F4       F5       F6       F7       +CONT
FLFACT   1        1.0                                                            DENS
FLFACT   2        .45                                                            MACH
FLFACT   3        .2       .16667   .15315   .14286   .12500   .11111   .10000   KFREQ
$FLUTTER SID      METHOD   DENS     MACH     RFREQ    IMETH    NVALUE   EPS
FLUTTER 30        KE       1        2        3        L        6
PARAM    LMODES   6
ENDDATA
```

Appendix A. – Convergence Behavior of the Generalized Aerodynamic Influence Coefficients.

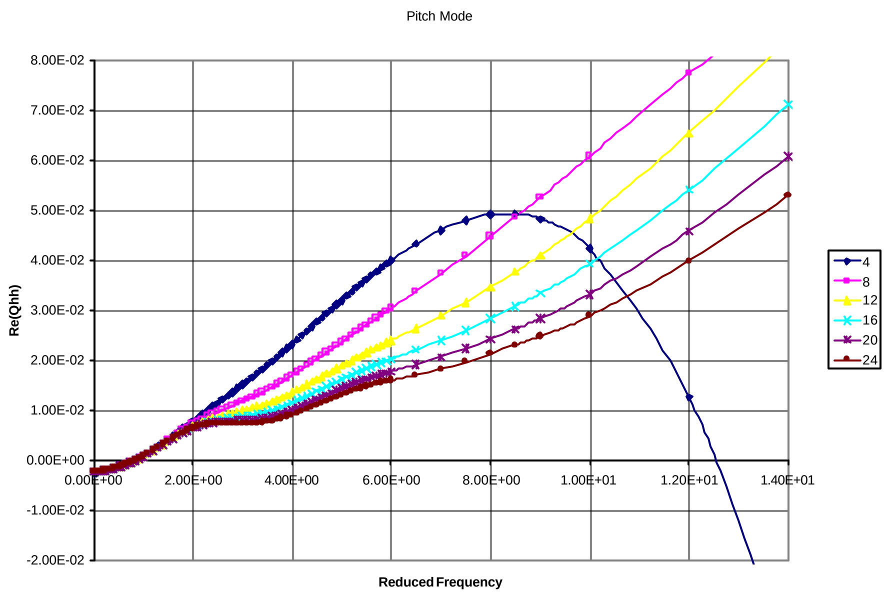

Figure A-1. - Subsonic Re($Q_{hh)}$ Matrix Convergence with Chordwise Aerodynamic elements as a function of Reduced Frequency.

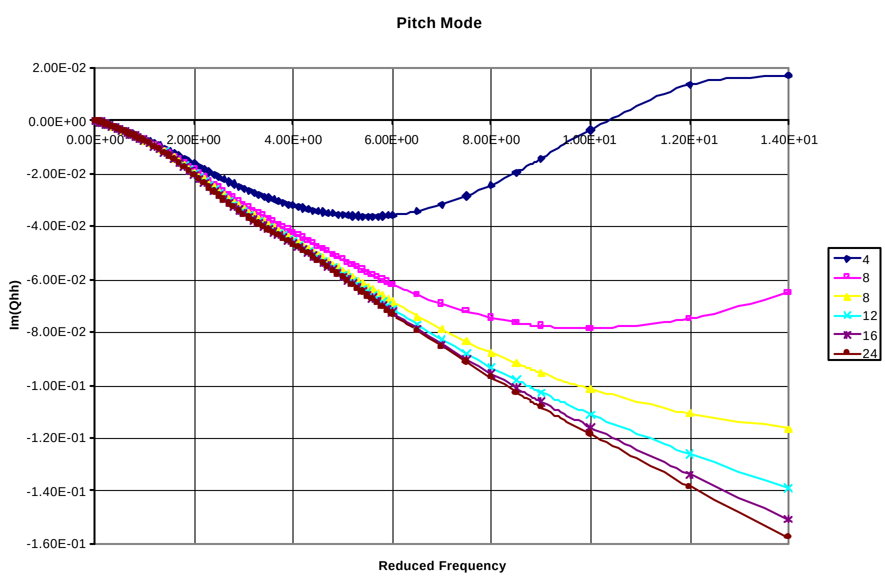

Figure A-2. - Subsonic Im(Q_{hh}) Matrix Convergence with Chordwise Aerodynamic elements as a function of Reduced Frequency.

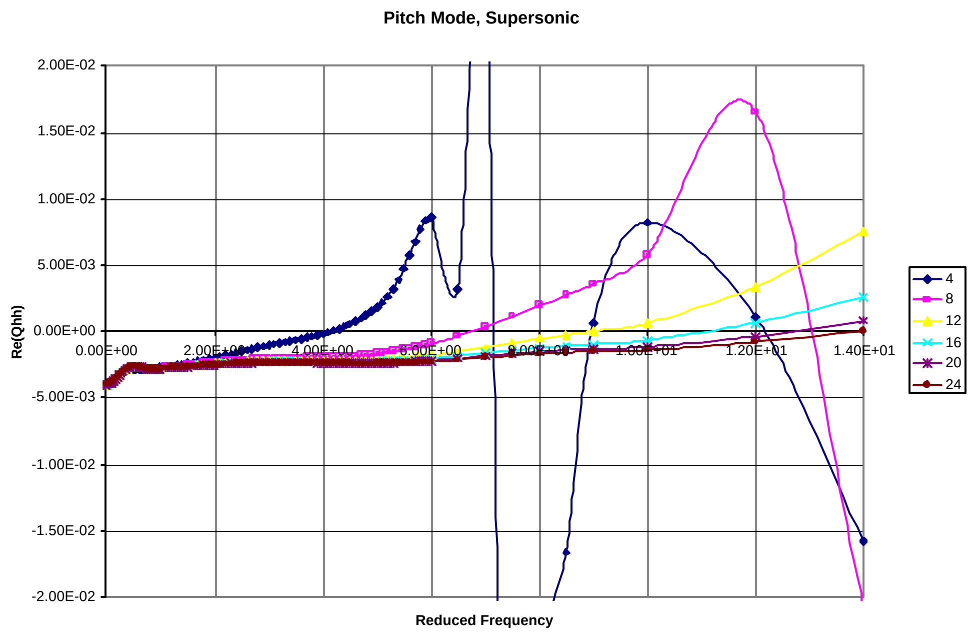

Figure A-3. - Supersonic Rl(Qhh) Matrix Convergence with Chordwise Aerodynamic elements as a function of Reduced Frequency.