Rotary-Wing Aeroelasticity: Current Status and Future Trends

Peretz P. Friedmann University of Michigan, Ann Arbor, Michigan 48109-2140

Introduction and Background

Concise Perspective and Previous Surveys

W HEN reviewing research in rotary-wing aeroelasticity (RWA), it is important to mention a few historical facts. The Wright brothers flew in 1903, and Sikorsky built and started flying the first operational helicopter, the R-4 or (VS-316), in 1942. The R-4 was a three-bladed helicopter with a rotor diameter of 11.6 m and was powered by a 185-hp engine. Thus, there is an initial gap of approximately four decades between fixed-wing and rotary-wing technologies. Therefore, it is not surprising that certain rotary-wing problems, particularly those pertaining to unsteady aerodynamics, are still not well understood. The situation is further compounded by the complexity of the vehicle when compared to fixed-wing aircraft. The field of RWA has been the most active area in aeroelasticity during the last three decades. This vigorous research activity has generated a considerable number of survey papers as well as several books that have been published on this topic.

These review papers, when considered in chronological order, provide a historical perspective on this evolving field.^{1–13} One of the first significant reviews of rotary-wing dynamic and aeroelastic problems was provided by Loewy,¹¹ where a wide range of dynamic problems was reviewed in considerable detail. A more limited survey emphasizing the role of unsteady aerodynamics and vibration problems in forward flight was presented by Dat.² Two comprehensive reviews of rotary-wing aeroelasticity were presented by Friedmann.^{3,4} In Ref. 3, a detailed chronological discussion of the flap-lag and coupled flap-lag-torsion problems in hover and forward flight was presented, emphasizing the inherently nonlinear nature of the hingeless-blade aeroelastic stability problem. The nonlinearities considered were geometrical nonlinearities because of moderate blade deflections. In Ref. 4, the role of unsteady aerodynamics, including dynamic stall, was examined, together with the treatment of nonlinear aeroelastic problems in forward flight. Finite element solutions to RWA problems were also considered, together with the treatment of coupled rotor-fuselage problems. Another detailed survey by Ormiston¹² discussed the aeroelasticity of hingeless and bearingless rotors, in hover, from an experimental and theoretical point of view.

Although aeroelastic stability plays an important role in the design of rotor systems, the aeroelastic response problem as represented by the rotorcraft vibration and dynamic loads prediction plays an even more critical role. Thus, two other surveys have dealt exclusively with vibration and its control in rotorcraft.^{14,15} These papers focus on the vibrations caused by the aeroelastic response of the rotor and the study of various passive, semi-active and active devices for controlling such vibrations.

Johnson^{9,10} has published a comprehensive review paper addressing both aeroelastic stability and vibration problems for advanced rotor systems. In a sequel⁵ to his previous review papers, Friedmann discussed the principal developments that have taken place between 1983–1987, emphasizing new methods for formulating aeroelastic problems, advances in treatment of the aeroelastic problem in forward flight, coupled rotor-fuselage analyses, structural blade modeling, structural optimization, and the use of active control for vibration reduction and stability augmentation.

A comprehensive report,¹³ which contains a detailed review of the theoretical and experimental development in the aeroelastic and aeromechanical stability of helicopters and tilt-rotor aircraft, carried out under U.S. Army/NASA sponsorship during the period 1967-1987 was prepared by Ormiston, Warmbrodt, Hodges, and Peters. Somewhat later, key ideas and developments in four specific areas-1) role of geometric nonlinearities in RWA, 2) structural modeling of composite blades, 3) coupled rotor-fuselage aeromechanical problems and their active control, and 4) higher harmonic control (HHC) for vibration reduction in rotorcraft-were considered by Friedmann.⁶ At the same time, Chopra¹ surveyed the state of the art in aeromechanical stability of helicopters, including pitch-flap, flaplag, coupled flap-lag-torsion, air and ground resonance. Advances in aeromechanical analysis of bearingless, circulation controlled, and composite rotors were also treated in this detailed paper. Perhaps the most comprehensive paper on RWA was written by Friedmann and Hodges.⁸ This paper contains close to 350 references and dwells on all of the important aspects of rotary-wing aeroelastic stability and response problems. The treatment is broad and comprehensive and is current up to 1991. A partial review of some recent developments can also be found in Ref. 7.



Peretz P. Friedmann is François-Xavier Bagnoud Professor of Aerospace Engineering in the Aerospace Engineering Department of the University of Michigan, Ann Arbor, and Director, FXB Center for Rotary and Fixed Wing Air Vehicle Design. He received his B.S. and M.S. degrees in Aeronautical Engineering from the Technion-Israel Institute of Technology and his Sc.D. (1972) in Aeronautics and Astronautics from the Massachusetts Institute of Technology (MIT). Prior to entering the academic world, Dr. Friedmann worked in Israel Aircraft Industries and was a Research Assistant at the Aeroelastic and Structures Laboratory at MIT. He has been with the University of Michigan since January 1999. Between 1972 and 1998 he was a Professor in the Mechanical and Aerospace Engineering Department of the University of California, Los Angeles. Between 1988 and 1991 he served as the Chairman of the department. Dr. Friedmann has been engaged in research on rotary-wing and fixed-wing aeroelasticity, active control of vibrations, hypersonic aeroelasticity, flutter suppression, structural dynamics, and structural optimization with aeroelastic constraints, and he has published extensively (more than 240 journal and conference papers). He was the recipient of the 1984 American Society of Mechanical Engineers (ASME) Structures and Materials Award, and he is a Fellow of AIAA and the American Helicopter Society. He was the recipient of the AIAA Structures, Structural Dynamics, and Materials (SDM) Award for 1996, and the AIAA SDM Lecture Award at the 38th SDM Conference, in 1997. He was the recipient of the Spirit of St. Louis Medal for 2003, given by ASME, and the ASME/Boeing Structures and Materials Award in 2004.

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In addition to the numerous papers dealing with the subject of this review, this topic is also described in a number of books. Among these, the most notable one is Johnson's¹⁶ monumental treatise on helicopter theory, which contains extensive, detailed, and useful material on aerodynamic, dynamic, and mathematical aspects of rotarywing aerodynamics, dynamics, and aeroelasticity. A more recent book¹⁷ treats several aeroelastic and structural dynamic problems in rotorcraft. Quite recently, Leishman¹⁸ has written an excellent book on helicopter aerodynamics, which contains good treatments of unsteady aerodynamics, rotor wake models, and dynamic stall.

Fundamental Differences Between Rotary-Wing and Fixed-Wing Aeroelasticity

The basic problem in fixed-wing aeroelasticity is the coupled bending-torsion problem, which is essentially a linear problem. The basic problem in rotary-wing aeroelasticity is the coupled flap lag torsion (CFLT) of an isolated blade, which is inherently nonlinear because of the geometric nonlinearities associated with moderate (or large) blade deflections that must be incorporated into the structural, inertia, and aerodynamic terms associated with this aeroelastic problem. A typical hingeless blade with an advanced geometry tip is shown in Fig. 1. The geometry of the basic CFLT problem is depicted in Fig. 2. The composite drawing depicted in Fig. 2 consists of three parts. The top part shows a view of the deformed blade projected on a plane perpendicular to the plane of rotation. The middle portion represents a view of the deformed blade projected on the plane of rotation. The bottom part represents the location of the blade cross section before and after deformation. For the RWA problem, coupling between bending out of the plane of rotation (flap), bending in the plane of rotation (lag) and torsion is critical, and neglect of one of these degrees of freedom can produce inaccurate and misleading results.

Rotary-wing aeroelastic problems can be separated in two regimes: hover and forward flight. In hover, the equations of motion have constant coefficients, whereas in forward flight the equations have periodic coefficients. The fundamentally nonlinear nature of RWA requires coupling between the aeroelastic problem and the flight condition of the entire helicopter as represented by its trim state. Two types of trim procedures, propulsive trim and wind-tunnel trim, have been used. The first trim procedure simulates straight-andlevel forward-flight conditions, as shown in Fig. 3, and the second trim procedure corresponds to the conditions experienced when testing the rotor on a support in the wind tunnel.⁴ An important aspect of rotary-wing aeroelasticity is the coupling between the trim state and the aeroelastic problem. This requires a simultaneous solution of the trim and aeroelastic problems. This coupling is often neglected in fixed-wing analysis. Aeroelastic stability boundaries can be obtained by linearizing equations of motion about the equilibrium position determined from a coupled trim-aeroelastic analysis. In hover, eigenanalysis is used to obtain the aeroelastic stability boundaries, and in forward flight aeroelastic stability is usually determined from Floquet theory.^{4,8}

The lead-lag degree of freedom, with its low aerodynamic and structural damping, is a critical degree of freedom in most rotarywing aeroelastic problems. Another important class of problems is coupled rotor-fuselage aeroelastic problem that is sometimes also denoted by the term aeromechanical problem. This problem involves coupling of the fuselage rigid-body degrees of freedom (primarily pitch and roll) with the blade degrees of freedom (primarily leadlag). The geometry depicting a typical coupled rotor-fuselage of a system is shown in Fig. 4. On the ground, the aeromechanical instability is called ground resonance, and in flight it is known as air



Fig. 1 Typical hingeless blade with advanced geometry tip.



Note: Blade cross section after deformation is not in the plane of the paper

Fig. 2 Undeformed and deformed blade configurations illustrating the geometrically nonlinear aspects of the basic coupled flap-lag torsional problem of an elastic blade.

resonance. Although active flutter suppression has not been an area of significant concern in RWA, active suppression of aeromechanical instabilities has received considerable attention. This is quite different from fixed-wing aeroelasticity, where there has been a sustained effort to deal with active flutter suppression.⁷

The aeroelastic response problem that manifests itself as blade loads, hub loads, or fuselage vibrations has a critical role for rotarywing vehicles. Vibration prediction and its control has been an area of intense activity. Modeling unsteady aerodynamic loads on the blade and the rotor is a major challenge. The combination of blade advancing and rotational speed is a source of complexity. At large advance ratios, many different flow regimes coexist: transonic flow with shock waves on the advancing blade and flow reversal and low-speed unsteady stall on the retreating blade. Time-varying wake geometry, which is an important source of unsteady loads, vibration, and noise, is excruciatingly complex. Computation of the unsteady free wake has been a major challenge, and it is essential for correct computation of vibrations and noise. Figure 5, taken from Ref. 19, depicts three free-wake calculations based upon three different freewake models. Rotor-fuselage interactional aerodynamics is another difficult problem. Clearly, these types of problems are unique to rotorcraft and are not encountered in fixed-wing aeroelasticity.

Objectives of This Paper

This paper describes developments in RWA that have taken place during the last decade, and thus it is a companion to Ref. 8. The paper treats both aeroelastic stability problems as well as the aeroelastic response problem by describing the principal research activities on the subjects listed here: 1) recent developments in the modeling of composite rotor blades undergoing moderate and large



Fig. 3 Schematic description of the helicopter in forward flight used for coupled trim/aeroelastic analysis (propulsive trim).



Fig. 4 Coupled rotor-fuselage dynamic system, with active control flaps shown on the blades.

deformation and their incorporation in a variety of aeroelastic stability and response studies; 2) modeling of nonlinear elastomeric lag dampers and their influence on aeromechanical and aeroelastic stability problems; 3) aeroelastic behavior of swept-tip rotors and correlation with experiments; 4) development and validation of comprehensive helicopter analysis codes; 5) relative aeroelastic characteristics of hingeless and bearingless rotors; 6) development of improved unsteady aerodynamic models, modeling of dynamic stall, and their incorporation in aeroelastic stability and response studies; and 7) active control of aeroelastic stability and response (i.e., vibration) in rotorcraft.

Not all of the topics just listed are treated with the same level of detail. The scope of the treatment of any particular topic depends on the amount of research available as well as overall space allocations within this survey. The subject of rotor loads prediction and correlation with experimental data has been deliberately excluded because it is best treated as a separate topic.

Composite Blade Models and Their Application

Structural dynamic and aeroelastic modeling of composite blades undergoing moderate or large deflections and their application to the study of hingeless, bearingless, and tilt-rotor blade aeroelasticity as well as coupled rotor-fuselage problems has been a particularly active area of research. Because of its importance, this research topic has also been addressed in several survey papers.^{8,20,21} The principal conclusions from this body of research are summarized next. The most important requirements for modeling composite helicopter blades are the capability to represent transverse shear deformation, cross-sectional warping, and elastic coupling caused by material anisotropy, in addition to an adequate representation of geometric nonlinearities. A most effective approach to modeling this complex beam problem is to split the basic three-dimensional, geometrically nonlinear elasticity problem into two separate problems: 1) a geometrically nonlinear, one-dimensional problem of a beam in the spanwise direction and 2) a two-dimensional linear elastic problem from which the warping and the cross-sectional properties at any spanwise station are determined. This two-stage approach was intuitively used by Kosmatka and Friedmann.²² A rigorous foundation for this approach was established by Atilgan and Hodges²³ using an asymptotic analysis. Later this approach was further refined by Cesnik et al.^{24,25} Finally, this approach was generalized, leading to a general cross-sectional modeling technique,²⁶ which is linked by a variational formulation to the one-dimensional beam theory. The authors denote this approach by the term "dimensional reduction."

The composite blade theories that are currently available can be separated into three groups:

1) The first group is theories that are based on dimensional reduction of the basic three-dimensional elasticity problem using a variational asymptotic method. In these theories, the spanwise beam problem is capable of handling large deformations and small strains. A separate finite element analysis is used to obtain the cross-sectional properties and the appropriate warping representation. The finite element cross-sectional analysis is capable of representing arbitrary cross sections, with anisotropic material properties, and a variety of wall thicknesses.^{23–27}

2) The second group is theories based on ad hoc decoupling between the geometrically nonlinear, spanwise one-dimensional beam theory, combined with a finite element cross-sectional analysis to determine the cross-sectional constants and warping. In these theories, the assumption of moderate blade deflections, based on an ordering scheme, is often used to simplify the spanwise one-dimensional beam model. The cross-sectional analysis is still capable of modeling anisotropic, arbitrary cross sections, with multicell construction and various wall thicknesses. This cross-sectional analysis is usually linear.^{22,28–31}

3) The last group is theories combining assumptions on thinwalled, single-cell or two-cell, composite cross sections, with a moderate deflection type of one-dimensional beam theory. In these theories, shear is included, and different types of warping functions can also be included. These theories usually provide useful and good approximations; however, they do not have the accuracy, generality, and logical appeal of either 1) or 2) type of theories.^{32–37}

The composite blade models just discussed have been used for a variety of applications. A representative, but incomplete, list of such applications is provided next: 1) determination of composite coupling coefficients, twist and deformation for composite beams or blades, for which experimental data are available,^{26,27,34,38,39} 2) free-vibration analysis of rotating composite blades,^{27,29,38}



Fig. 5 Qualitative features of three different free-wake models at an advance ratio of $\mu = 0.1$. Wake geometry: a) free-wake model, b) Johnson (modified Scully) model, and c) RotorCraft model.

3) aeroelastic behavior of composite hingeless and bearingless rotor blades in hover and forward flight, $^{29-31,33,37,40-42}$ 4) air and ground resonance of helicopters with elastically tailored composite blades, 35 and 5) tilt-rotor aeroelastic stability and response with elastically coupled composite rotor blades. $^{43-46}$

Some results taken from Ref. 31 are shown here to illustrate the effect of composite blade construction on the aeroelastic stability of a typical composite hingeless helicopter blade in hover and forward flight. The fundamental, coupled rotating natural frequencies of the blade in lag, flap, and torsion, respectively, are $\omega_{L1} = 0.765$, $\omega_{F1} = 1.096$, and $\omega_{T1} = 3.356$, when the ply orientation is zero. The four-bladed hingeless rotor operates at a average weight coefficient $C_W = 0.005$ and a Lock number of $\gamma = 5.0$, with solidity $\sigma = 0.076$.

The undeformed element coordinate system for the swept-tip composite blade is shown in Fig. 6. The blade model belongs to category 2) described earlier in this section. The finite element nodal degrees of freedom for the spanwise, moderate deflection beam model are shown in Fig. 7. The two-cell composite cross section, which is analyzed by a separate finite element cross-sectional analysis, is depicted in Fig. 8. Stability of the lag degree of freedom is illustrated in Fig. 9. Usually this is the degree of freedom tends to become unstable in hover. The effect of the composite construction is illustrated for a straight blade where the ply angle Λ_v in the middle vertical and the inner half of the rear vertical wall is changed between $-90 \text{ deg} < \Lambda_v < 90 \text{ deg}$. As evident from the real part of this root locus plot, the change in ply angle can influence blade stability (i.e., damping) substantially in both a beneficial and detrimental manner. However, it is insufficient to destabilize the blade. The imaginary part of the plot shows the changes in the frequency of the aeroelastic lag mode as a result of changes in ply orientation. Numerous results can be found in Ref. 31, where it is shown that combination of ply orientation and tip sweep can slightly destabilize the blade in hover. Figure 10 shows the effect of ply orientation on hingeless blade stability in forward flight, at an advance ratio of $\mu = 0.30$. For forward flight stability, information is extracted from the linearized perturbation equations about the coupled trim-aeroelastic



Fig. 6 Undeformed blade coordinate system, showing typical elements of the blade span.

response in forward flight, using Floquet theory.⁸ The real part of the characteristic exponent is an indicator of the damping in the lag degree of freedom. Again, the ply angle Λ_v is varied between $-90 \text{ deg} < \Lambda_v < 90 \text{ deg}$. Although damping levels change substantially, ply orientation is not sufficient to destabilize the blade for this particular case. Other results from Ref. 31 indicate that blade response and loads can be changed significantly by changing ply orientation.

These results, and similar results obtained in the various studies mentioned in this section, clearly indicate that composite blade



Fig. 7 Finite element nodal degrees of freedom.



Fig. 8 Two-cell composite cross section, including ply orientations Λ_{ν} and Λ_{h} in the vertical and horizontal walls, respectively.



Fig. 9 Root locus for the first lag mode.

construction has considerable potential for aeroelastic tailoring leading to enhanced blade stability and reduced vibratory loading in the rotor. However, this remarkable potential has not been significantly exploited by the helicopter manufacturers. Most modern helicopter blades are built of composites, primarily for their excellent fatigue characteristics, which result in unlimited blade life. Thus, composite blades have a large advantage over the older metal blades that had to be replaced after a few thousand hours of operation.



Fig. 10 Characteristic exponent for the fundamental lag mode at an advance ratio of $\mu = 0.30$.

Modeling of Elastomeric Lag Dampers and Their Incorporation in Aeroelastic Analyses

The effect of lag dampers on aeroelastic and aeromechanical stability of hingeless and bearingless rotored helicopters has always been an important area of endeavor. During the last decade, important advances in this area have been made. The most important recent developments have focused on the analytical and experimental modeling of the nonlinear properties of elastomeric lag dampers.^{47–54}

The body of research in this area consists of two different approaches. Gandhi and Chopra^{49,50} have developed a nonlinear elastomeric damper model, based on a combination of linear and nonlinear springs and dashpots. The damper is characterized by a nonlinear constitutive differential equation. This approach is capable of capturing, partially, the viscoelastic, hysteretic, energy-dissipating properties of elastomeric lag dampers, as well as dependence on operating conditions and steady-state lag angles. The damper model is incorporated in the ground resonance problem of an articulated rotor and in the air resonance problem of a hingeless rotor, in hover. The blades have only flap and lag degrees of freedom, the fuselage has pitch and roll, and the damper states are appended to the equations of motion. The results show that modeling of the dampers is important for stability prediction. However, the paper⁴⁹ made no attempt to connect the damper model with any real elastomeric damper that is in actual use. Subsequently, in Ref. 50, a slightly improved model of the damper was incorporated in an aeroelastic and aeromechanical analysis of a three-bladed bearingless rotor in forward flight. Both shaft-fixed and shaft-free aeroelastic behavior were examined, and the role of the damper was found to be important. However, the damper and its behavior were not related to an actual elastomeric damper used in a particular helicopter; an application that would have further clarified the methodology.

A much more fundamental and comprehensive approach to the modeling of nonlinear lag dampers is presented in Refs. 47, 48, 53, and 54. In the first⁵⁴ of these two studies, a nonlinear anelastic-displacement-field (ADF) damper model, based on accurate three-dimensional material modeling and irreversible thermodynamics, was developed from basic principles. Material nonlinearities are introduced by nonlinear functions that describe the dependence of the unrelaxed and relaxed material moduli and the anelastic strain rate, on the instantaneous total and anelastic strains. The parameters that characterize the nonlinear material behavior are identified through harmonic-strain-controlled experimental tests. Nonlinear finite element equations are obtained in terms of the resulting ADF parameters.

In the second paper⁵³ for a simplified case, corresponding to pure shear behavior, two coupled partial differential equations are obtained: one describes motion, and the second governs creep



Fig. 11 Offset hinged spring-restrained hingeless blade model, coupled with an elastomeric lag damper model.



Fig. 12 Hysteretic characteristics of damper force vs lag angle for ADF damper model.

evolution in time. The parameters required for the model implementation are obtained from suitable material characterization tests. Damper behavior is modeled by a finite element method and combined with a three-degree-of-freedom offset hinged, springrestrained blade model, as shown in Fig. 11. Nonlinear lag damper equations are coupled with the blade equations and are solved simultaneously. Nonlinear equations are linearized about the steady-state response solution. Blade stability in forward flight is obtained from Floquet theory. The silicon rubber damper was modeled with a single finite element. The nonlinear behavior of the elastomeric damper has a significant effect on lag mode stability in hover and forward flight. Damper response in forward flight depends on blade loading and advance ratio. A good indicator of the success of this model to predict the hysteretic behavior of lag dampers is illustrated by Fig. 12. This figure shows the nondimensional lag damper force vs lag angle for several advance ratios: $\mu = 0, 0.05, 0.20, \text{ and}$ 0.40, for fixed blade loading. The static lag angle decreases initially from hover to a minimum at an advance ratio of 0.20, and subsequently it increases. Dynamic amplitudes increase steadily with advance ratio. Note that the ADF damper model predicts substantial variations in the area and aspect ratio of damper hysteresis loops with advance ratio. The $\mu = 0.40$ loop displays the characteristic asymmetry shown by elastomeric materials undergoing harmonic excitation with a static bias. The elastomeric lag dampers add both stiffness and damping to the rotor system.

In another sequel to this study,⁴⁷ the model was extended to include friction-type elements. These elements consist of frictiondamping and linear-spring elements, which are added in parallel to the original ADF model. The new model represents a substantial improvement over the basic ADF model, described earlier in this section. Thus, the peak error between predicted and experimentally determined material complex moduli was reduced from 72 to 18% for the storage modulus and from 90 to 10% for the loss modulus. This model represents a modification of the first generation ADF model⁵⁴ to capture elastomeric behavior at low strain amplitude (where material nonlinearity is most significant) and over a range of frequencies from quasi-static to approximately 2/rev. This model also captured important effects such as dual-frequency stress-strain hysteresis loops.

In the final paper in this series,⁴⁸ the elastomeric material described in Ref. 47 is used to design an elastomeric damper, which is coupled with an isolated rotor aeroelastic analysis. The damper state equations are coupled directly to the rotor-blade equations. Isolated rotor stability, in-plane response, and damper loads in hover and forward flight are examined. The paper demonstrates effectively that the model developed is suitable for elastomeric lag damper preliminary design.

Aeroelastic Behavior of Swept-Tip Rotors

Rotor blades with swept tips, which imply both sweep and anhedral of the blade-tip region, shown in Fig. 1, experience bendingtorsion and bending-axial coupling effects caused by sweep and anhedral. Swept tips influence blade dynamics because they are located at regions of high dynamic pressure and relatively large elastic displacements. Swept tips enhance rotor performance and are effective for reducing aerodynamic noise and blade vibrations. Most modern helicopter blades have swept tips, which sometimes are also denoted by the term advanced geometry tips. Therefore the study of aeroelastic behavior of rotor blades with advanced geometry tips has received considerable attention during the past decade.^{29-31,38,42,55-59} In Ref. 56, isotropic swept-tip rotors with advanced geometry tips have been treated. Moderate deflection beam theory is coupled with unsteady transonic-small-disturbance theory (TSD) to calculate the unsteady transonic flow, three-dimensional effects, and shock waves on the advancing blade tips. The time history of the angle-of-attack distributions, obtained from the rotor dynamic code, is used as input to the finite difference TSD code. The nonuniform inflow distribution over the rotor disk is calculated using a free-wake model. The coupling between the various analyses is carried out in a somewhat ad hoc manner, which causes the results to be suitable primarily for response calculations, and thus blade stability was not considered.

The aeroelastic behavior of swept-tip composite blades was considered in a number of detailed studies.^{29–31,42} The principal objective of these studies was the development of a practical, efficient structural dynamic model for swept-tip rotor blades undergoing moderate deflections. Subsequently, the model was used



Fig. 13 Comparison of CAMRAD II calculations with three inflow models and measured hover regressing mode damping vs collective pitch angle, swept-tip blade with 2-deg precone hub, sweep 30 deg aft, and no anhedral.

to determine the effect of tip sweep and composite blade construction on the aerodynamic stability of swept-tip rotor blades in hover and forward flight. Aeroelastic blade response and blade loads in forward flight were also considered in these studies. The principal goal of these studies was the examination of the structural coupling effects. Therefore, the aerodynamic loads used were based upon quasi-steady Greenberg theory combined with uniform inflow. Unsteady loads were formulated using a general implicit formulation,⁵⁵ which permitted the replacement of these loads by compressible unsteady loads combined with a free-wake analysis in studies that were conducted later.^{60,61} Results obtained in these studies^{29–31,42} show that combinations of tip sweep and composite constructions can substantially affect both blade stability, response, and loads.

The aeroelastic stability of straight and swept-tip rotor blades in hover and forward flight was studied both experimentally and analytically in two recent comprehensive papers.^{57,58} Two Mach-scaled rotors, with a diameter of 7.5 ft, having both straight and swept-tip blades, were tested in hover and forward flight. The purpose of the tests was to provide hover and forward-flight data for comparison with analytical models. Data from the tests were compared with calculations from a comprehensive rotor code, CAMRAD II.^{62,63} Correlations between test and calculations for regressing lag mode damping are shown in Fig. 13 from Ref. 58. The correlation between theory and experiment is very good. The aerodynamic loads were calculated with three different inflow models: no dynamic inflow, Pit and Peters dynamic inflow, and momentum theory dynamic inflow. The differences as a result of these inflow models are small.

For forward flight, the correlation between the test results for regressing lag mode damping and the experimental data is not as good as in hover as evident from Fig. 14, taken from Ref. 58. The addition of dynamic inflow increases the damping; thus, the results without dynamic inflow are closer to the experimental data. Subsequently in Ref. 57, sensitivity studies, using parameter variations in aerodynamic center, center of gravity, blade mass, torque offsets, etc., were conducted. With these parameter variations, modest improvements between theory and test results for the swept-tip rotors in forward flight were obtained.

The studies considered in this section indicate that the accurate modeling of aeroelastic stability and response of swept-tip rotors still represents a challenge, and good correlations with experimental data are not easily achieved.

Development and Validation of Comprehensive Helicopter Analysis Codes

The development and validation of comprehensive helicopter analysis codes such as CAMRAD II,^{62,63} 2GCHAS,^{52,64–69}



Fig. 14 Comparison with swept-tip rotor of measured forward-flight regressing-lag mode damping, CAMRAD II periodic solution without dynamic inflow and with dynamic inflow, 0-deg shaft angle, 6-deg collective pitch, 0-deg precone hub, 30-deg aft tip sweep, and no anhedral.

RDYNE,⁷⁰ COPTER,⁷¹ UMARC,⁷² and CAMRAD/JA⁷³ has been another topic that has received considerable attention. Among these codes, the two most advanced, CAMRAD II and 2GCHAS, have considerable capabilities that permit the treatment of complex configurations. The CAMRAD II code takes advantage of the area of multibody dynamics to facilitate the treatment of complex configurations.^{74,75}

Among the various comprehensive helicopter analysis codes, CAMRAD II is perhaps the most widely used, both in the United States as well as Europe and Japan. The code has been slightly more successful than its competitors in correlating with experimental data. The 2GCHAS code has also undergone considerable validation during the last five years, and overall the correlations indicate generally satisfactory predictive capability for a fairly wide range of rotorcraft problems. A modified and improved version of the 2GCHAS code has recently become available; it is denoted by the name Rotorcraft Comprehensive Analysis Code. In addition to considerable improvements that enhance its computational efficiency and reduce the run times required, the code has the added advantage of being able to run on PC platforms using the Linux operating system.⁷⁶

The UMARC code developed at the University of Maryland has also enjoyed considerable success, as students who graduated have taken the code with them and started using it in an industrial setting.

Aeroelastic Characteristics of Hingeless and Bearingless Rotors

During the past three decades, the helicopter industry in the United States and abroad has invested a very substantial amount of resources in the development of production hingeless and bearingless rotor systems. Hingeless rotored helicopters, such as the MBB BO-105, and the Westland Lynx have been in production for almost 25 years. However, successful bearingless rotored helicopters have gone into production only during the last decade. Typical examples are the MD-900 Explorer,⁷⁷ the Comanche bearingless main rotor (BMR),⁷⁸ the Eurocopter EC135.⁷⁹ Also, the MD900 and the Comanche have five-bladed rotors, whereas the EC135 is four bladed. This is an indication that BMR technology has matured in the past decade, and substantial gains in the understanding of aeroelastic and aeromechanical aspects of these rotors have been made. It is also fair to say that the improved understanding of hingeless rotor behavior has made significant contributions toward improved understanding of BMRs. This section highlights research carried out during the past decade that has contributed towards the fundamental understanding of hingeless and bearingless rotor aeroelastic behavior.

A number of theoretical studies dealing with the physical coupling mechanism present in hingeless rotored helicopters air and ground resonance in hover⁸⁰ and forward flight⁸¹ have clarified the coupling mechanism between body and blade modes and the effect of forward flight on air resonance. An aeroelastic analysis methodology for BMR helicopters in hover and forward flight was developed by Gandhi and Chopra.⁸² The approach correlated reasonably well with shaft-fixed hover stability data obtained in tests, and then it was used to predict shaft-free stability in forward flight. These theoretical studies were useful; however, work done by the helicopter industry, which is discussed next, reveals in a more comprehensive manner the advances made in the dynamic design of hingeless and bearingless rotor systems.

An insightful study by Weller⁸³ provides a comparison of the aeromechanical stability characteristics, in hover, for two models of conventionally designed soft-in-plane main rotors. One model is a bearingless configuration, simulating the Bell helicopter M680 main rotor. The second model is a hingeless rotor similar to the MBB BO-105 main rotor. The purpose of the study was to compare the test data obtained from the two models, identify their respective aeromechanical stability characteristics and determine the design features that have a primary effect on the air and ground resonance behavior in hover.

To understand better the relative aeroelastic characteristics of hingeless vs BMRs, illustrations describing their operation are relevant. The geometry of a typical hingeless rotor was depicted in Fig. 1. The bearingless rotor with an elastomeric damper is more complicated and is shown in Fig. 15, taken from Ref. 78. In a bearingless rotor, each blade is connected to the hub center by a flexural arm called a flexbeam, which bends in lag, flap and twists by undergoing torsional deformation. The flexbeam usually has an inboard portion that establishes a virtual lag hinge, whereas the outboard portion is shaped so as to yield a torsionally soft region for relatively large control inputs. A structurally stiff external cuff encloses the flexbeam and transmits control inputs to the blade, which are applied by the pitch link. The cuff is bolted to the blade and the flexbeam at its outboard end, as shown in Fig. 15. At the inboard end of the cuff, a shear-restraint mechanism, called a snubber, connects the cuff to the flexbeam. The shear restraint transmits both flapwise and lagwise shears from the cuff to the flexure. The snubber housing contains elastomeric shear pads that are used to augment structural damping in the lead-lag or in-plane direction, and thus this combination is called an elastomeric lag damper, also shown in Fig. 15. To further clarify the operation of the bearingless hub and damper combination, consider Fig. 16 taken from Ref. 79. The top portion of Fig. 16 is the view of the blade from the top, and the bottom part is a section through the flexbeam, damper and control (right part) as well as a section through the damper (left side).

The mechanism, whereby the inplane bending of the flexbeam in lead lag produces displacement and damping, is illustrated by Fig. 17, also from Ref. 79. Figure 17 shows the displacement between the cuff and flexbeam at the location of the damper caused by



Fig. 15 Description of the Comanche bearingless main rotor, including both elastomeric and Fluidlastic[®] damper configurations.



Fig. 16 Design of EC135 bearingless rotor geometry, flexbeam, and elastomeric lag damper.



Fig. 17 Chordwise kinematics showing cuff and in-plane blade bending (top), which determine the modal displacement and damping of the elastomeric lag damper (bottom).

in-plane bending as well as the lead-lag damping that is produced. The shear stiffness of the damper depends on material and geometrical properties, environmental conditions (temperature), axial loads, as well as displacement amplitudes. The modeling of elastomeric lag dampers, described in an earlier section, is an important issue in bearingless rotor design. Finally, the cuff flexbeam combination represents a structural element with a redundant load path, which has to be modeled carefully.⁸⁴

In Ref. 83, two Froude-scaled models, one hingeless and one bearingless, were tested. Although the paper does not explicitly state the scale used, a comparison of the model radius with the MBB BO-105 blade radius reveals that it is a 1:4 scale version of the production rotor, and it is reasonable to assume that the bearingless rotor was scaled in the same manner. The rotors were tested on the Advanced Rotorcraft Experimental Dynamics system, which can provide body pitch and roll degrees of freedom at both low and high thrust conditions. The results obtained indicate that the hingelessrotor concept offers better stability margins at moderate to high thrust conditions because of its aeroelastic characteristics; thus, the hingeless rotor is more stable at 1 g thrust and above. For low thrust conditions, however, the bearingless rotor is better because of its larger structural damping caused by the elastomeric lag damper. In these comparisons, it is also important to keep in mind that the hingeless rotor had no lag damper, and its damping was caused by its inherent structural damping. In a sequel⁸⁵ to this study,⁸³ Weller conducted a very careful ex-

In a sequel⁸⁵ to this study,⁸³ Weller conducted a very careful experimental examination of a somewhat larger model BMR with a diameter of 9.37 ft. The BMR used in Ref. 83 had a diameter of 7.95 ft. This was also a four-bladed rotor, with an elastomeric lag damper similar to that considered earlier.⁸³ Several parameters were changed during the test, such as fundamental flap frequency, blade



Fig. 18 Hover air resonance of the minimum flight weight configuration with elastomeric dampers at 8-deg collective pitch.

built in cone and sweep angles, pitch link inclination, flexbeam prepitch, and shear restraint to flexure attachment. The extensive results show that modest changes in these parameters provided only very minor improvements in the aeromechanical stability margins. The two studies mentioned^{83,85} contain a large amount of valuable experimental data, but very limited correlation with analytical models is provided.

An outstanding study is Ref. 78, which describes in detail the aeroelastic stability wind-tunnel testing of the Comanche BMR and presents correlations with an analytical model. This BMR configuration is depicted in Fig. 15. A series of wind-tunnel tests were performed on a $\frac{1}{6}$ Froude-scaled model of the RAH-66 Comanche BMR at the Boeing V/STOL wind tunnel. The tests had two objectives: 1) establish the aeromechanical stability characteristics of the coupled rotor-fuselage system, and 2) correlate the experimental data with analytical stability predictions so that the methodology can be used with confidence for the full-scale aircraft. An initial test of the rotor with elastomeric dampers, shown in Fig. 15, uncovered a limit cycle instability. This instability manifested itself for the minimum flight weight configuration. Figure 18, taken from Ref. 78, depicts the frequency and damping of the coupled rotor-body system with elastomeric snubber/dampers. The presence of the body degrees of freedom and their coupling with the blade degrees of freedom modifies significantly the dynamic characteristics compared to the isolated rotor case. A frequency coalescence between the lagregressing and the flap-regressing body roll mode now exists. Near this coalescence, the damping is low, and a limit-cycle oscillation occurs at the regressing lag frequency. Closer examination of this nonlinear problem, presented in the paper⁷⁸ revealed that this problem might also be present when flying with the prototype flight weight. A decision was made to replace the elastomeric snubber/damper by a Fluidlastic[®] snubber/damper, which is also shown in Fig. 15. The Fluidlastic snubber/damper is similar to the elastomeric dampers except that it includes a chamber within the flat elements, which is filled with silicone fluid to provide the blade lead-lag damping. As the elastomeric elements that constitute the wall of the chamber flex in shear, the fluid is forced to flow around a rigid diverter protruding into the fluid, thereby generating a damping force.



Fig. 19 Hover air resonance at 9-deg collective with Fluidlastic damper.

Further study revealed that nonlinearities in the stiffness and loss factor of the elastomeric snubber/dampers were the cause of this limit-cycle behavior. As shown in Ref. 78, the stiffness of the Fluidlastic damper is nearly linear, and using it eliminates the limit-cycle instability. Figure 19 shows the hover air response characteristics of the prototype flight weight configuration with the Fluidlastic dampers at 9-deg collective. The test data for both frequency and damping are also compared with analytical results obtained from the UMARC/B code, which is a Boeing modified version of UMARC.⁷² The correlations between the results for the code in both hover and forward flight are quite good.

The aeromechanical design aspects of another production BMR that is flying on the European EC135 are described in Ref. 79. The feature of this BMR equipped with an elastomeric lag damper are shown in Fig. 16. This rotor has excellent damping margins throughout its operation envelope. Modal damping for this rotor in level flight is shown in Fig. 20. The dots are from the flight test, and the solid line is the result of a calculation performed by CAMRAD II. The agreement between theory and test is good. The damping amounts to approximately 2.5% in the rotating system.

It is remarkable that all three production BMR systems have been developed during the past decade. This clearly indicates that the state of the art of BMR aeroelastic design has made substantial advances in this period.

Improved Unsteady Aerodynamic Models and Modeling of Dynamic Stall

The accurate representation of the unsteady aerodynamic loading environment on rotorcraft plays a critical role in the prediction of aeroelastic stability and response. The representation of the nonlinear structural portion as well as the inertial loading for the rotarywing aeroelastic problem can be done accurately by using finite elements and multibody techniques. However, the prediction of the unsteady aerodynamic environment presents a major challenge, and in some cases large errors in the calculation of the unsteady aerodynamic loads cannot be avoided. When dealing with the unsteady aerodynamic loads, it is convenient to distinguish between two separate cases: approximate unsteady aerodynamic models for attached-



Fig. 20 Regressing-lag mode damping in forward flight and comparison with CAMRAD II.

flow and dynamic-stall models. These topics together with their application to the rotary-wing aeroelastic problem are discussed next.

Approximate Unsteady Aerodynamic Models and Their Application to Aeroelastic Problems

An useful unsteady aerodynamic model that provides approximate unsteady aerodynamic loads in the time domain was developed by Peters and his associates.^{86,87} The finite-state inflow model is an approximate theory that provides global rotor-disk downwash under unsteady flight conditions, and it is different from airfoil theory that produces local downwash. The finite-state inflow theory is an exact representation of the three-dimensional incompressible potential flow equations of the wake. The wake is assumed to be a cylinder in hover and a skewed cylinder in forward flight. However, the model also incorporates the important effect of wake contraction. Detailed descriptions of this theory can also be found in Refs. 88 and 89. This theory has been employed in several aeroelastic stability studies in hover and forward flight.

A typical application of this theory to the aeroelastic stability of composite rotor blades in hover can be found in Ref. 90, which integrates geometrically exact nonlinear beam theory and the generalized dynamic wake theory within a finite element framework. The theory is applied to several examples, including limited correlation with experimental data.

In another study,⁹¹ the three-dimensional finite-state wake model⁸⁸ is applied to hingeless-rotor aeromechanical stability in hover and forward flight. A very interesting aspect of this study is the correlation with experimental data for an aeromechanical stability problem in which the regressing-lag mode couples with fuselage roll to produce an instability. Figure 21, taken from Ref. 91, shows the regressing-lag mode damping as a function of rotor speed (rpm). The dots are the experimental data, the solid line is for M = 1, which represents an earlier version of this theory,92 utilizing a single harmonic component, and the dashed line corresponds to the threedimensional finite-state wake model with nine harmonic wake components (M = 9). Clearly, for this particular case, there is practically no difference between the finite-state wake model and dynamic inflow. The good correlation between the theory and the experimental data evident for the regressing-lag mode is not replicated by the other modes, as can be seen by examining the other results in this study. However, in all of the cases considered, there is practically no difference between the results based on dynamic inflow and those based on finite-state wake theory with nine harmonics.

Another, related, interesting study conducted recently⁹³ attempts to enhance the efficiency of the finite-state wake model by using aerodynamic eigenmodes to develop reduced order models for



Fig. 21 Regressing-lag mode damping with increasing rotor speed at 0-deg blade collective pitch.

rotorcraft. This technique, pioneered by Dowell, has proven itself quite effective for fixed-wing and turbomachinery flows. The study concluded that the finite-state wake model is very compact, and its effectiveness cannot be enhanced by reduced-order modeling.

One of the shortcomings of the finite-state wake model is its inability to model blade vortex interaction (BVI), which is an important source of rotor vibratory loads at lower advance ratios. To model BVI effects, free-wake models, similar to those depicted in Fig. 5, are required. An efficient rotor free-wake model was developed by Bagai and Leishman.⁹⁴ This wake model has been incorporated into the UMARC code and has been used in numerous studies since then.

Another new and useful approximate unsteady aerodynamic theory that was developed to model two-dimensional, compressible loads for an airfoil/flap combination in unsteady freestream is presented in Refs. 95 and 96. Aerodynamic loads are obtained in the frequency domain for an airfoil/flap combination using the doubletlattice method. Subsequently, the loads are approximated in the frequency domain as rational functions of the Laplace variable using a least-squares fit to the unsteady aerodynamic loads. Transformation to the time domain yields a state-space model for the unsteady aerodynamic loads. An important feature of this approximate unsteady aerodynamic model is its ability to model the hinge moment on actively controlled flaps used for vibration reduction, which are discussed later in this paper.

Dynamic Stall and Its Incorporation into Aeroelastic Analyses

Dynamic stall is a strongly nonlinear unsteady aerodynamic effect, which plays a major role in aeroelastic stability and response calculations. This topic is reviewed in detail in a new book by Leishman,¹⁸ in Chapter 9. Dynamic stall is associated with the retreating blade and borders on the reversed flow region, as shown in Fig. 22. For such conditions, the angle of attack of the blade cross section can be very large. Although the torsional response of the blade is relatively low under normal conditions, at the flight envelope boundary, where dynamic stall effects are pronounced, large transient-torsional excursion can be excited, accompanied by low negative damping in pitch. This, in turn, generates excessive control and blade vibratory loads, which impose speed and load limitations on the rotor as a whole. It can also cause stall flutter. Because of its importance, dynamic stall has been the subject of a large number of studies that have led to an improved understanding of this complex aerodynamic effect.^{4,5,8,18} Dynamic stall is usually incorporated in rotary-wing aeroelastic analysis using semi-empirical models. These models have a number of common features. They are intended to incorporate two-dimensional, nonlinear, airfoil unsteady aerodynamic effects in analytical studies in the time domain, and they are suitable for stepwise numerical integration in time. The semi-empirical nature of the model is because of a number of free



Fig. 22 Schematic illustration of reversed flow region and dynamicstall region.

parameters that are determined by fitting the theoretical expressions to the experimental data, obtained from oscillating airfoil tests.

During the past two decades, two primary semi-empirical dynamic-stall models have emerged:

1) The ONERA dynamic-stall model describes the unsteady airfoil behavior in both attached flow and during dynamic stall using a set of nonlinear differential equations. Since its initial derivation in 1984, the model has undergone revisions and improvement. The most recent version of this model was documented by Petot.97 The coefficients in the equations of this model are determined by parameter identification from experimental measurements on oscillating airfoils. The model requires 22 empirical coefficients. Figure 23 shows typical hysteretic lift and moment coefficients computed with the ONERA dynamic-stall model, for a NACA 0012 airfoil, at M = 0.379, k = 0.075, and time-varying angle of attack $\alpha = 10.3 \text{ deg} + 8.1 \sin \omega t$. A more recent modification of this model has been proposed by Truong.98 The modification contains a Hopf bifurcation model and some three-dimensional corrections, it requires 18 coefficients. However, this modification displays nonphysical oscillations in the unloading part of the hysteresis loop depicted in Fig. 23, and therefore it has not gained the wide acceptance awarded to the earlier model.

2) The Leishman–Beddoes model⁹⁹ is capable of representing the unsteady lift, pitching moment, and drag characteristics of an airfoil undergoing dynamic stall. This model consists of three distinct components: 1) an attached flow model for the unsteady linear airloads, 2) a separated flow model for the nonlinear airloads, and 3) a dynamic-stall model for the leading-edge vortex-induced airloads. The model contains a rigorous representation of compressibility in the attached flow part of the model, using compressible indicial response functions. The treatment of nonlinear aerodynamic effects associated with separated flows are derived from the Kirchoff-Helmholtz model to define an effective separation point that can be generalized empirically. The model uses relatively few empirical constants, with all but four derived from static airfoil data.¹⁸ This model is also undergoing continuous improvement. The most recent one was introduced in Ref. 100. This dynamic-stall model has been embedded in the 2GCHAS⁶⁸ as well as the UMARC⁷² codes.

The dynamic-stall models just described briefly have been used in a number of aeroelastic stability and response studies. The ONERA dynamic-stall model has been employed in several aeroelastic stability studies. In Ref. 101, the effect of dynamic stall on isolated blade flap-lag stability is considered. The nonlinear equations of blade



Fig. 23 Typical hysteretic lift and moment coefficients computed with the ONERA dynamic-stall model.

motion of a hingeless rotor modeled as the offset hinged spring restrained-blade model, with coupled flap-lag dynamics for each blade, are combined with the dynamic-stall model. The nonlinear equations of blade motion are perturbed about a periodic forced response, and the damping is evaluated by Floquet eigenanalysis. The damping is correlated with experimental data, and the correlation is not very satisfactory. The reason for this discrepancy is probably because perturbation of the equations in the presence of dynamic stall is a questionable approach. A more effective approach would have been to integrate the equations of motion in the time domain and extract the damping information from the response curves for the appropriate rotor degrees of freedom. In a sequel to this study, presented in Ref. 102, the authors consider a hingeless rotor with fully coupled flap-lag torsional dynamics (two mode representation for each degree of freedom), and the aerodynamic loads are obtained by a combining the ONERA dynamic-stall model with a finite-state wake model. The correlation between analysis and test is fair. Figure 24, taken from this study, depicts the influence of different levels of aerodynamic modeling: 1) blade element theory with quasi-steady stall, 2) the same theory but with dynamic stall, and 3) dynamic stall and finite-state wake theory. The figure shows the lag damping for a three-bladed hingeless rotor at two shaft angles α_s , over the advance ratio range $0 < \mu < 0.60$, and a collective pitch setting of $\theta_0 = 3$ deg. Figure 24a shows the rotor operating at low thrust; all three models are reasonably close to experimental data. Figure 24b depicts a large negative shaft angle $\alpha_s = -16$ deg, and damping decreases with advance ratio. This is a low-thrust case, and dynamic-stall and static-stall theories are close. Dynamic stall and the wake model improve the agreement at low μ ($\mu < 0.20$); however, these theories fail to predict the loss of damping at the higher advance ratio. This is somewhat surprising because one would expect improved correlation with increased advance ratio because dynamic-stall effects are known to be important at high advance ratios. It is plausible that the reason for this discrepancy is again a result of the method used for computing the damping, which was mentioned earlier in connection to Ref. 101.

Aeroelastic stability studies involving the Leishman dynamicstall model were conducted by Torok and Chopra.¹⁰³ The effect of



Fig. 24 Aerodynamic modeling effects on the lag damping, for a threebladed hingeless rotor-blade model.

flow separation and dynamic stall on rotor-lag damping in highspeed flight is quite significant.

Finally, recently a European study group has completed a monumental report that compares several dynamic-stall models, in their ability to predict rotor behavior in the presence of dynamic stall.¹⁰⁴ Seven different stall models were compared in this study, and the predictions were compared with detailed measurements conducted on a model rotor in a wind tunnel. The study has identified threedimensional phenomena as being significant. Clearly, this effect is beyond the reach of present two-dimensional semi-empirical models. This effect produces an unexpected increase in lift and moment at 0.7R blade span. The authors speculate that progress in computational aerodynamics, as applied to rotors, might lead to a qualitative explanation of these phenomena.

Active Control of Aeroelastic Stability and Response (i.e., Vibration) in Rotorcraft

The desire to develop rotorcraft having a "jet smooth" ride has shifted the emphasis in the area of vibration alleviation (i.e., reduction of aeroelastic response) from traditional passive means of vibration reduction such as vibration absorbers and isolators to active control strategies.¹⁰⁵ Stringent requirements on vibration levels strive for vertical accelerations below 0.05g at most fuselage locations. These requirements imply that helicopter manufacturers might be willing to tolerate the expense associated with an active control system that operates in the rotating frame, that is, the rotor. Once such a control system is present, it can also be used for additional objectives such as noise reduction, performance enhancement, and stabilization of aeroelastic phenomena. This section summarizes and highlights recent accomplishments in two areas: stabilization of aeroelastic and aeromechanical phenomena and vibration reduction in rotorcraft using active controls.

Stabilization of Aeromechanical and Aeroelastic Phenomena by Active Controls

Some of the previous research in this area has been reviewed and discussed in Ref. 8. One of the most comprehensive studies on air resonance suppression, in hover and forward flight, using blade pitch control was carried out by Takahashi and Friedmann.¹⁰⁶ The model consisted of a coupled rotor-fuselage system representing a four-bladed hingeless rotor attached to a rigid fuselage, as depicted in Fig. 4, with pitch-and-roll degrees of freedom. The controller operated through a conventional swash plate that introduced the same pitch input to all of the blades. The controller design was based on an optimal state estimator combined with optimal feedback gains. Optimal loop shapes were designed using the loop transfer recovery approach. The outcome of this design process resulted in a simple controller that used a single roll-rate measurement in the body (nonrotating frame) and suppressed air resonance by using a sine and a cosine swash-plate input. The controller was shown to stabilize the system throughout a wide range of loading conditions and forward flight speeds, with pitch inputs of 3 deg or less.

More recently, Weller¹⁰⁷ conducted an experimental program to demonstrate the benefits of applying active rotor control techniques to improve the aeromechanical stability characteristics of a BMR model. This model of the BMR was identical to that tested earlier.85 Apparently the author was unaware of the analytical work done earlier in this field.¹⁰⁶ He also seemed reluctant to use modern control techniques, and much of this work was done either experimentally or by using an electrohydraulic simulator. Pitch-and-roll velocities and accelerations were measured and transformed into pylon position and velocity information, which was fed back using a fixed-gain relationship to produce cyclic swash-plate commands θ_c and θ_s so as to increase the damping of the aeromechanical problem. Results indicate the pylon position feedback could increase damping by 1%, at most. However, destabilizing trends at high thrust were present. Pylon velocity feedback was superior to position feedback, and the destabilizing trends were reduced. This result is entirely consistent with the findings of Ref. 106. Thus, it illustrates that experimental trial-and-error approaches to active control, without the benefits of analytical simulation and the physical understanding it produces, can substantially reduce the effectiveness of such studies.

Finally, air resonance, which is an aeromechanical instability, is a mild instability as opposed to flutter, which is an explosive instability. Stabilizing flutter in rotorcraft using active control has not received serious consideration because it was deemed to be impractical. A partial exception to this philosophy is represented by a recent experimental study that has evaluated an approach called generalized predictive control (GPC) to augment aeroelastic stability of a tiltrotor operating in the airplane mode.¹⁰⁸ The tests were performed on a $\frac{1}{5}$ th scale, semispan aeroelastic model of the V-22, designed and built by Bell in 1981. A special tiltrotor research testbed called the Wing and Rotor Aeroelastic Testing System was used in the Langley Transonic Dynamics Tunnel. The case considered is the airplane mode, to be precise, a propeller-whirl type of instability was controlled, and strictly speaking, this is again a fixed-wing type of aeroelastic instability. Nevertheless, because of its innovative nature, this was an important contribution. It was demonstrated that using three inputs in the stationary swashplate the GPC approach was highly effective in increasing the stability of the critical wing mode, when wing responses were used as feedback. For most cases, the damping in wing beam mode was increased from less than 0.5% of critical to over 3% of critical, throughout the entire flight regime.

Recently, Celi¹⁰⁹ has written an interesting paper on stabilization of a blade with a severed pitch link, using a trailing-edge flap, schematically depicted in Fig. 25. This study is on the boundary between aeroelasticity and reconfigurable controls. The failure of the pitch link causes the blade to be free floating and uncontrollable. The study indicates that the trailing-edge flap is capable of correcting the catastrophic consequences of the pitch link failure by trimming the blade, through an optimization-based trim procedure. Unfortunately, flap deflections of 18–22 deg are required, and these combined with actuation power requirements, which were not addressed in this study, could prevent the practical implementation of such a concept.

Vibration Reduction in Rotorcraft

As mentioned earlier, vibration reduction in rotorcraft is essentially the control of the aeroelastic response problem. Active con-



Fig. 25 Blade with severed pitch link and actively controlled trailingedge flap.



Fig. 26 Single- or dual-ACF configuration used for vibration reduction.

trol approaches to rotorcraft vibration reduction are perceived to be a requirement so that rotorcraft can experience vibration levels comparable to fixed-wing transport aircraft. During the past 25 years, several approaches to active vibration control in rotorcraft have emerged.¹⁰⁵ The first approach developed was HHC. The controller applies pitch inputs through a conventional swashplate. All blades experience the same inputs, and the vibratory aerodynamic loads are modified at their source, before they propagate into the fuselage. A more promising alternative is individual blade control (IBC), where time-varying pitch is introduced directly in the rotating reference frame. The IBC approach can be implemented using three different techniques. One can oscillate the entire blade in pitch by actuating it at the root; this approach was used in the earliest implementation of the IBC methodology. Alternatively, a small partial-span trailing-edge flap, shown in Fig. 26, can be actuated on the blade; this approach is sometimes called the actively controlled flap (ACF). An even more effective approach is to use two flaps; this configuration, also shown in Fig. 26, is denoted as the dual-flap configuration. A third implementation twists the entire blade by embedding piezoelectric fibers; this approach is known as the active twist rotor (ATR); the blade structure for this configuration is shown schematically in Fig. 27. All approaches just mentioned control vibrations in the rotating frame. An alternative approach sometimes known as active control of structural response (ACSR) is aimed at vibrations in the fuselage, or the fixed frame, as illustrated in Fig. 28. In this approach, stiff actuators introduce small-amplitude excitation between the rotor and the fuselage, such that the sum of the response of the airframe at specified locations, because of rotor loads and the excitation caused by controls, is reduced to a minimum.

Among various active approaches to vibration reduction, only the ACSR system has been actually installed on a production helicopter,

the EH101, built by a European partnership between Westland and Agusta. All of the other systems have been tested in wind tunnels. The HHC approach and IBC scheme with root actuation have been flight tested,¹⁰⁵ whereas the ACF was supposed to be flight tested in early 2003 on a MD-900 Explorer, as part of the Smart Rotor demonstration program, funded by DARPA. However, instead of the flight test, the full-scale rotor with a piezoelectrically actuated ACF system was tested on a whirl tower in Fall 2003. The changes in the objectives of the program were caused by a combination of technical problems associated with the actuation system and availability of funding.

The practical importance of active vibration control in rotorcraft has resulted in a large number of papers on this topic during the last decade, and to do justice to this topic would require a separate survey paper. Thus, only the most significant papers in this category will be described in this section.

Among the various active control approaches, IBC implemented using the actively controlled flap appears to be the most promising concept, and therefore it has been extensively pursued. Furthermore, the ACF has also considerable potential for noise reduction and performance enhancement. A number of studies have established the remarkable potential of the ACF, implemented either in the singleflap or dual-flap configuration (Fig. 26), for vibration reduction using a flexible blade model, with coupled flap-lag torsional dynamics and modified Theodorsen aerodynamics that include the effect of



Fig. 27 ATR spar structure with active laminates containing piezoelectric fibers.

time-varying freestream.^{110–113} Milgram et al.¹¹⁴ have developed an analytical simulation incorporating an unsteady compressible aerodynamic model. The aeroelastic model was developed using the comprehensive analysis code UMARC. Experimental results from wind-tunnel tests of the ACF were also presented;¹¹⁵ the purpose of these early studies was to demonstrate the feasibility and effectiveness of this new approach to vibration control.

The need for an improved aeroelastic simulation model for the flap-blade combination led to the development of new and improved models based on a compressible time-domain unsteady aerody-namic model. This simulation capability could accommodate three different flap configurations, including dual flaps. Detailed vibration reduction studies from this model were presented in Refs. 95 and 116–118.

Subsequently, this model was improved by adding a free-wake model to the time-domain unsteady compressible theory.^{60,61,118,119} The resulting comprehensive simulation model facilitated the examination of two distinctly different flight regimes in which vibrations are reduced using the ACF: a high-speed flight regime, where advance ratio effects are dominant and the influence of the free wake is limited, and low or moderate advance-ratio regime where BVI are important. These studies have clearly demonstrated that vibration reduction at low advance ratios ($\mu = 0.15$) is a more demanding control task because of the presence of BVI, than vibration reduction at high speeds of $\mu = 0.30$ or higher.

During this time period, very valuable experimental results on the practical implementation of the ACF and its application to fundamental vibration reduction in the open-loop mode, on a two-bladed rotor, were reported by Fulton and Ormiston.¹²⁰ This model rotor was not representative of any particular full-scale configuration. The availability of these results permitted a comparison between the comprehensive simulation developed⁶¹ and the experimental data obtained by Fulton and Ormiston.¹²⁰ To illustrate the vibration reduction capability of the actively controlled flap and the reliability of the simulation model, some typical results obtained in Refs. 61 and 119 are presented next.

The results are for a four-bladed hingeless rotor that has properties that resemble the MBB BO-105 rotor. Figure 29 shows the baseline and controlled vibratory hub shears and moments, with compressible unsteady aerodynamics (referred to as RFA aerodynamics) and a free-wake model, at an advance ratio $\mu = 0.15$, where BVI is important. Figure 30 depicts similar results at a higher advance ratio $\mu = 0.30$. Two important observations are relevant: 1) the vibratory loads in the presence of BVI are approximately four times higher than those at $\mu = 0.30$; and 2) although the actively controlled flap is quite effective in reducing vibrations at both advance ratios, its



Fig. 28 Coupled rotor/flexible fuselage model using ACSR platform and actuators.



Fig. 29 Simultaneous reduction of 4/rev hub shears and moments ($\mu = 0.15$), RFA aerodynamics.



Fig. 30 Simultaneous reduction of 4/rev hub shears and moments ($\mu = 0.30$), RFA aerodynamics.



Fig. 31 Flap deflection history at advance ratios $\mu = 0.15$ and 0.30, RFA aerodynamics.

performance in the presence of BVI is not as good as in the higher advance ratio regime. Figure 31 depicts the flap deflections required for vibration reduction at these two advance ratios; the maximum flap deflection required for the alleviation of BVI effects can exceed 15 deg. Thus, BVI vibration alleviation is more demanding than vibrations at high speed. All of the results presented were for a flap operating in the closed-loop mode using a control law described in Refs. 61 and 119.



Fig. 32 Variation of 2/rev flapwise bending moment with elevon phase (760 rpm, $\mu = 0.20$), RFA aerodynamics.



Fig. 33 Variation of 3/rev flapwise bending moment with elevon phase (760 rpm, $\mu = 0.20$), RFA aerodynamics.

The simulation capability described here was also validated by comparing it with experimental data obtained in Ref. 120 for the open-loop operation of the flap, and the correlation in most cases is quite good. Typical results obtained in these correlation studies, which serve as a validation of the simulation capability,^{61,119} are shown in Figs. 32 and 33. The experiment results were obtained on a two-bladed¹²⁰ rotor at an advance ratio of $\mu = 0.20$. The rotor was excited by flap inputs at 2, 3, 4, and 5/rev; the magnitude of the flap input was $\delta_f = 5$ deg. The root flapping moment of the blade in the rotating system was measured. These flapping moments were also simulated by the code. Results are shown in Fig. 32 for the 2/rev excitation and in Fig. 33 for the 3/rev excitation. The two blades tested were not identical, and therefore each plot contains two sets of experimental data, one for blade 1 and another for blade 2, respectively. The simulations were conducted for an average blade, and the results are shown by the triangles in Figs. 32 and 33. Clearly, the agreement between the simulation and the test is quite good.

An experimental demonstration on the feasibility of using piezoelectrically actuated flaps for vibration reduction in forward flight was conducted by Koratkar and Chopra.^{121,122} The rotor was tested in the University of Maryland wind tunnel. It was a four-bladed Mach scaled bearingless rotor resembling a Bell-412; the scale was approximately $\frac{1}{7}$ th of full scale. The flaps were actuated by piezoelectric benders. When operating in the closed-loop mode, a neuralnetwork controller was used. Reference 121 describes primarily hover and open-loop tests, whereas Ref. 122 describes the closedloop tests in forward flight. The largest flap deflections recorded were in the range of 4 deg < $|\delta_f| < 6$ deg for components introduced with frequency of 1, 2, 3, 4, and 5/rev. With this control authority, 70–90% reduction in the vibratory loads was obtained in the advance ratio range of $0.10 < \mu < 0.30$ for relatively low thrust coefficient. Comparisons between the experimental data and computer simulation were not presented in the paper.

A number of recent papers have also examined several issues associated with the practical implementation of ACF systems to the vibration reduction problem. One important problem is that of excessive flap deflections, which is evident in Fig. 31, where the maximum flap deflections can reach 15 deg. These angles are larger than angles that can be achieved with active or smart materials-based actuation. Furthermore, in most practical cases, flap authority will have to be limited to 3-4 deg, so as to avoid interfering with the handling qualities of the helicopter. This has raised the issue of flap saturation that was studied in Ref. 123, by examining three different approaches: 1) clipping of flap deflection, when the angles reach a prescribed value; 2) uniform scaling such that flap input harmonics never exceed a limiting value; and 3) an iterative, automatic, adjustment of the control weighting matrix, so that the flap deflection is properly constrained. It was found that the first two approaches completely eliminated the effectiveness of the ACF as vibration reduction device. However, the third approach proved itself to be a very effective approach to limiting saturation. Overall, a hub vibration reduction penalty of only 10%, compared to the unconstrained flap, was obtained, with flap deflections limited to 4 deg, thus leaving the vibration reduction capacity of the flap virtually intact.

Two additional studies^{124,125} have considered the capability of single- and dual-ACF systems to alleviate vibrations caused by dynamic stall at high advance ratios. Furthermore, the effect of freeplay on the vibration reduction effectiveness of the ACF was also studied in Ref. 125. The effect of dynamic stall was incorporated in the simulation¹²⁴ by using the ONERA dynamic-stall model and combining it with the unsteady aerodynamics, described in Refs. 61 and 119. Another important ingredient added in this study was the drag caused by the flap deflection.¹²⁴ Using a conventional control algorithm, employed in most HHC and IBC studies, it was shown that the ACF flap is very successful in alleviating vibra-tions caused by dynamic stall.¹²⁴ The vibration reduction obtained is shown in Fig. 34 for both a single-flap as well as a dual-flap configuration. In both cases, saturation limits, limiting flap deflections to $-4 \deg < \delta_f < 4 \deg$ were imposed. Again, the dual-flap configuration was more effective in vibration reduction than the single-flap configuration. Vibratory hub shears were reduced by 70-80% and vibratory hub moments by 60-85%. It was also found that the drag penalty caused by flap deflection can increase power requirements by 1-2% of rotor power. In a sequel to this study, the effects of a freeplay type of nonlinearity on the vibration reduction effectiveness of the ACF system was also considered,125 and small amounts of freeplay caused only a small degradation in the effectiveness of the flap, as a vibration reduction device.

Numerous other studies on vibration reduction using actively controlled flaps were carried out. Reference 126 was one of the



Fig. 34 Vibration reduction with dynamic stall using single- and dualflap configurations, with saturation limits, $\mu = 0.35$.

fundamental papers in this group. A detailed survey paper by Chopra¹²⁷ reviews many studies that have attempted to combine piezoelectric actuation with trailing-edge flaps for vibration reduction. Other studies¹²⁸ have also considered magnetostrictive actuation for the flap. Straub and his coworkers¹²⁹ have simulated vibration reduction by ACF system using the comprehensive analysis code CAMRAD II; these studies were in support of the development of a full-scale rotor test with piezoelectrically actuated flaps.

As evident from survey papers^{127,130} dealing with adaptive materials-based actuation for vibration reduction applications, there has been a concern about the force and stroke-producing capability of piezoelectric-based actuation. Furthermore, in many cases, such actuators have been demonstrated using small, scaled models of rotors. Therefore, it is important to pay attention to aeroelastic scaling considerations¹³¹ before assuming that actuators that perform well in a small-scale test will also work on a full scale rotor.

The concern whether adaptive materials are adequate for implementing the ACF for full-scale configurations has motivated research on alternative actuation schemes. A potentially interesting concept is the electromagnetic actuator for individual blade control.¹³² The electromagnetic actuator was combined with a trailing-edge control surface, called the HeliflapTM, which appears to be suitable for active rotor control applications. The device proved itself to be rugged, compact with no external linkages and no moving parts except the flap itself, and it has good force, deflection, and frequency characteristics. The amplitude and frequency were controlled by modulation of the electrical current to the actuators. The device was bench tested and whirl tested on a full-scale OH-58 rotor at 81% operating rpm and low collective. Flap deflections of ±6 deg at frequencies of 4.4/rev were achieved. This device appears to be promising.

Another successful combination between individual blade control and adaptive materials-based actuation is the ATR shown in Fig. 27, which was developed jointly by Massachusetts Institute of Technology, the Army, and NASA Langley Research Center.^{133,134} The first of these studies describes the aeroelastic modeling of the ATR using a simple flap-torsion model of the active twist blade, which is intended for approximate studies, and is implemented in a code called PETRA. The results are compared with results from CAMRAD II. The vibration results based on CAMRAD II predicted large vibration reduction in the 4/rev vertical hub shear, for less than 2 deg maximum twist. The second paper describes the correlation between the structural dynamic model developed for the ATR blade, bench tests, and rotating tests. The prediction of the blade torsional load in hover is within 20%.

This research has culminated in recent tests that demonstrated the effectiveness of the concept by reducing vibrations at low and relatively high advance ratios on a four-bladed, aeroelastically scaled active twist rotor.¹³⁵ The rotor was designed to be tested in the heavy-gas medium of the NASA Langley Transonic Dynamics Wind Tunnel. The rotor was operated in the open-loop mode, and the active twist inputs consisted of 3, 4, and 5/rev components. Depending on the flight conditions, the active twist angles at the blade tip were between 1.1 and 1.4 deg, and the advance ratio range was $0.14 < \mu < 0.367$. Fixed system load reduction was between 60–95%. Subsequently, these results were compared with analytical predictions obtained from CAMRAD II, and reasonable agreement was found at the low-advance-ratio range. It was also shown in this study that power requirement for operating the ATR was approximately 1% of rotor power.¹³⁶

There was also considerable activity in vibration reduction in the fixed system, namely, the fuselage, using active control. A very comprehensive study of vibration reduction using ACSR, employing a sophisticated simulation model was completed by Cribbs and Friedmann.¹³⁷ This study was the first to use rotor fuselage interactional aerodynamics, combined with a free-wake model, and a three-dimensional finite element model of the fuselage. Several control algorithms were tested, and excellent vibration reduction was demonstrated in the simulations.

Reference 137 was an extension of Ref. 138, which used a much simpler aerodynamic model that did not include the effect of

rotor/fuselage interactional aerodynamics, nor the effect of the free wake. Comparison of the results obtained from these studies clearly demonstrates the importance of using refined (i.e., accurate) aerodynamic models when simulating the behavior of an ACSR system on a computer. It is important to mention that both studies^{137,138} use centrally located actuators, placed beneath the main rotor, depicted in Fig. 28. A different approach to locating the actuators for active control of airframe vibrations was considered in Ref. 139. In this study, an optimization procedure was used to distribute the actuators at optimal locations throughout the fuselage. A hybrid optimization methodology was formulated to simultaneously determine optimal actuator placement and control actions. The results obtained were compared to a scheme based on centrally located actuators, and it was shown that distributed actuators can control the dominant airframe modes better, and achieve somewhat greater reduction with less control effort. This study however was not based on aerodynamic excitation generated in the code. Instead two sets of external loads, one at the main rotor hub and another at the horizontal tail, were used to excite the airframe model. The paper did not clarify the connection between this loading system and the actual unsteady aerodynamic loading environment present on the rotor.

Conclusions

The research activity in rotary-wing aeroelasticity during the last decade has been vigorous, innovative, and it was the most active research area in aeroelasticity. This is quite impressive when one recognizes that rotary wing vehicles represent 4-6% of the total aerospace business. Despite the fact that a number of problems in rotorcraft aeroelasticity and aeromechanics have existed for a considerable amount of time, good fundamental understanding and reliable solutions are often obtained slowly and with considerable difficulty. The most important milestones during this period have been the following: 1) the development of three new helicopters with bearingless main rotors, two in production (EC135, MD-900) and one (Comanche) on the verge of production; 2) development of methods based on first principles, for characterizing elastomeric lag dampers; 3) development of reliable techniques for modeling of composite main rotor blades with advanced geometry tips; 4) development of effective active control methods for vibration reduction in rotorcraft, and in particular, the approach based on the actively controlled trailing-edge flap; and 5) improved understanding of semi-empirical dynamic-stall models and their incorporation in rotary-wing aeroelastic stability and response analyses.

Future Trends

Despite the perception that exists in some circles that RWA is mature, and new areas whose description is characterized by fashionable collections of current buzzwords are more important, the author predicts that RWA and aeromechanics will continue to prosper because a number of fundamental problems are still not understood and are not amenable to reliable computer simulations. The need to build better rotorcraft with low vibration levels, low noise, and enhanced performance will continue to motivate researchers in RWA and aeromechanics to develop innovative approaches for the solution of these problems.

An area that is critical for both RWA as well as for the design of better rotorcraft is development of good coupling methods between finite element based structural dynamic models of blades and computational fluid mechanics for rotors. This relatively new field known as computational aeroelasticity⁷ is in its infancy for fixed-wing vehicles and has received very little attention in RWA. Development of this field can have major payoffs in the prediction of rotor loading, which has been a weakness in rotorcraft design. Much work has been done on rotorcraft vibration reduction using adaptive materials-based actuation. However, if the electromagnetically actuated flap¹³² lives up to its potential, it could supplant the adaptive materials based actuation as a possible approach for actuating the ACF. This will facilitate the development of active rotors, with dual trailing-edge flaps, that can be used for vibration reduction, noise alleviation, and performance enhancement. Persistent research on the simulation of dynamic stall by direct solution of the Navier–Stokes equations has the potential for replacing semi-empirical two-dimensional dynamic stall models by three-dimensional models based on first principles. Such a simulation capability will also lay the groundwork for developing active methods for controlling dynamic stall. However, achieving some of these objectives might take 5–10 years because solution of these problems in a short time frame is not feasible.

Development of new configurations of small unmanned rotorcraft could provide the impetus for innovative designs and concepts. Scaling issues in the design of such vehicles could also be very important. Development of new techniques applicable to such small vehicles could eventually have a significant impact on the larger manned vehicles.

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