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# Investigation on Transonic Correction Methods for Unsteady Aerodynamics and Aeroelastic Analyses

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**This paper presents an expedient transonic correction technique to compute unsteady pressure distributions and aeroelastic stability in the transonic flow regime. The transonic correction procedure here is an improvement of the downwash weighting method proposed previously by several authors. The previous downwash weighting methods could provide pressure and/or force corrections to some extent by applying different weighting methods on the lifting-surface self-induced downwash resulting from aeroelastic structural displacements or prescribed motions. However, the resulting pressure/force solutions were often found to be inconsistent, because they all failed to include the proper transonic unsteady and out-of-phase effects. Our improved downwash correction method is a rational formulation to include proper transonic effects, as this formulation is based on a successive kernel expansion procedure established in accord with the formal pressure-downwash relation. Accordingly, the developed transonic correction procedure is a proper and rational one that is expected to yield more consistent aeroelastic solutions. This procedure is now a fully developed program, known as the transonic weighting aerodynamic influence coefficient procedure in the ZAERO software system, or ZTAW. Computed results by ZTAW for the unsteady pressures and aeroelastic stability boundaries for four selected wing planforms (AGARD 445.6, F-5, LANN, Lessing wings) are found to be in good agreement with measured data. In contrast to the computational-fluid-dynamics-based methods of computational aeroelasticity, the present procedure is proven to be far more computationally efficient and industrially viable while yielding comparable aeroelastic solutions.**

## I. Introduction

THE application of discrete element kernel function methods are limited to purely subsonic or supersonic flows, because the governing equations over which the methods were developed are based on a linearized unsteady potential flow hypothesis. However, the aeroelastic behavior of an aircraft is typically critical in the transonic flight regime, in which nonlinear phenomena related to embedded moving shock waves and viscosity play an important role in aeroelastic stability. As discussed by Ashley [1], the shock wave movement and strength profoundly affect the flexure-torsion flutter mechanism. One consequence of this behavior is the so-called transonic dip. This phenomenon is characterized by a decrease of the slope of the flutter speed plot as a function of the Mach number, when compared with the same plot obtained from a linear aeroelastic analysis. Therefore, it is necessary to pay special attention to the flutter phenomenon under these circumstances. This is especially significant because most modern aircraft fly in transonic flow conditions.

One of the feasible alternatives to analyzing the aeroelastic stability in nonlinear flow conditions is the use of time-accurate computational fluid dynamic (CFD) solutions of the nonlinear fluid equations coupled with structural dynamic representation of the vehicle. Another approach is obtained by wind-tunnel testing of aeroelastic models under transonic flow conditions. However, wind-tunnel testing for aeroelastic investigation regarding flutter-boundary computation is not usual, because this class of experiments involves expensive models and high operational costs. The other way to evaluate the transonic aeroelastic behavior is from flutter flight testing, which is the most hazardous and expensive option in terms of operational costs. This approach may be used either for experimental flutter-boundary identification or to verify the aeroelastic subcritical aerodynamic damping at specific flight-envelope points to validate aeroelastic numerical models.

There have been several attempts to solve the transonic aeroelastic problem using combined procedures that relate linear models to measured data for the correction of unsteady linear aerodynamic models. Reviews on correction techniques were presented by Palacios et al. [2] and Silva [3], describing the most employed methods concerning transonic flutter prediction via combined procedures. Such approaches are named here as mixed procedures. The purpose of such procedures is to correct the linear aerodynamic models to take into account nonlinear effects that are not predictable by the linearized potential-based equations of the fluid flow.

The methodologies for the solution of the transonic aeroelastic problem based on mixed procedures are also referred to as semi-empirical corrections. These corrections can be performed by the multiplication, addition, or whole replacement of the aerodynamic influence coefficient (AIC) matrix. This approach is an adequate tool for engineering applications, because the methodology employed is less expensive than direct use of CFD techniques. The correction techniques, which have been applied to unsteady loading calculation for static or dynamic stability analysis, were classified by Silva [3] in four major groups: force-matching methods (Yates [4], Giesing et al. [5], Zwaan [6], Pitt and Goodman [7], Brink-Spalink and Bruns [8]); pressure-matching methods (Rodden and Revell [9], Bergh and Tijdeman [10], Bergh and Zwaan [11], Luber and Schmid [12], Baker [13], Baker et al. [14], Jadic et al. [15,16]); Dau-Garner type

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procedures (Garner [17], Dau [18], SenGupta [19], and Yonemoto [20]); and modal aerodynamic influence coefficient (MAIC) matrix replacement (Suciu et al. [21], Liu et al. [22], and Chen et al. [23]).

The main idea of the first procedure is to match reference nonlinear forces and moments, which may be obtained from experiments or accurate CFD solutions of the nonlinear flow governing equations. In this case, nonlinearities such as pressure jumps due to shock waves and viscous effects are embedded in such reference quantities. The second form to proceed with the correction is the matching of the pressures taken as reference conditions. The same nonlinearities are present in the reference conditions: in this case, pressure distributions instead of loads. The Dau–Garner class of methods [18,20] are procedures in which the unsteadiness of the resulting nonlinear corrected pressures are computed based on steady nonlinear information with the aid of semi-empirical relations. Finally, the MAIC procedures [21,23] consist of generating a modal aerodynamic influence coefficient matrix referred to measured or computed nonlinear pressures or loading due to given modal displacements of the lifting surface. This new matrix is then substituted in the aeroelastic equations of motion, in which the generalized unsteady aerodynamic forces are related to the associated modal displacements of the lifting surface. The aeroelastic analysis is performed taking into account transonic flow effects embedded in the new modal aerodynamic influence coefficient matrix.

Most of the aforementioned correction procedures employ steady state reference data, which may be loads or pressures. Some of them are based on the computation of corrected unsteady pressures from semi-empirical relations or with the aid of computational fluid dynamics simulations. After analyzing the correction procedures reviewed by Silva [3], some conclusions arose regarding their performance for transonic flow computation. For example, it could be noted that the force-matching method developed by Giesing et al. [5] does not guarantee that the modified pressure distribution, resulting from the corrected AIC matrix, will be the same as the experimental distribution or the distribution computed with the nonlinear method. In the presence of shock waves, this pressure distortion can severely modify the aeroelastic behavior. Furthermore, the nature of the transonic dip depends on the shock wave characteristics with regard to its positioning and strength. Nevertheless, at least the modified pressure distribution will guarantee that the sectional lift and moment will be the same. But this fact is not sufficient to yield a complete restoration of the nonlinear quasi-steady flow shock behavior as it would not be the same as the reference condition regarding the shock dynamics contribution.

The best way to preserve the reference nonlinear flow conditions regarding the shock structure is the direct matching of pressures by the use of a weighting operator. However, pressure-based weighting methods [9–12] present problems with regard to the evaluation of the pressure ratios necessary for the computation of the correction factors. If in any case there is a zero or very low pressure value in the denominator, the correction will present a tendency to blow up or yield very high correction-factor values, resulting in a poor conditioning of the resulting corrected AIC matrix.

An alternative to reformulate the adjustment of the pressures, to avoid the numerical problems mentioned already, is to correct the downwash vector. This approach consists of the modification of the control-point displacement vector (downwash) to satisfy a given reference pressure distribution. The advantage of this method, when compared with the pressures correction, is that the weighting-factor computation depends on a linear system equation solution instead of a simple pressure ratio. For this reason, even though there are null displacements or pressures given as reference conditions, the solution of the linear system of equations will not result in weighting factors being excessively large as happens in the pressure-matching procedures at those conditions.

Pitt and Goodman [7] and McCain [24] explore downwash correction methodologies, procedures that are based on the postmultiplication of a modified AIC matrix, which relate strip loads to its degrees of freedom. Pitt and Goodman's [7] procedure is a force-matching method because the reference conditions are strip forces and moments. The choice of the correction of the downwash is based on considerations highlighted by McCain [24]. That author discussed in his work that the postmultiplication weighting operators allow modification of both the real and imaginary parts of the downwash, thus changing the pressure amplitudes and phase angles.

After analytically reviewing such correction methods, it was concluded that the combination of the matching of the pressures through the downwash weighting presents good robustness and the preservation of the mean steady flow nonlinear characteristics. These features indicate that this approach may be developed for the prediction of the approximate unsteady transonic aerodynamic loading. Downwash correction procedures based on steady reference pressures may be a good option for the discrete element aerodynamic approximate modeling for transonic flow. However, the procedure based on weighting functions computed from steady pressures does not present good results with regard to the approximation of the imaginary counterpart of the nonlinear unsteady pressures [3].

The objective of the present work is to study correction techniques applied to unsteady transonic flow computation. Downwash weighting procedures are robust, inexpensive in terms of computational costs, and are compatible with the physics of the problem. They also present the advantage of providing a simple way to modify the pressures obtained from the linear theory by the replacement of externally computed or measured pressure distributions. These procedures will be investigated using either steady or unsteady pressures. An extension of this formulation will also be developed to circumvent the problem of the wrong pressure phase computation, without the use of nonlinear unsteady pressures taken as reference conditions for the correction procedure. The methodology of the present work is summarized next:

1) Understand the small-disturbance nonlinear transonic flow characteristic with regard to its behavior when subjected to small-disturbance conditions.

2) Evaluate the downwash correction techniques using either steady or unsteady reference pressures, and also understand its behavior based on the nature of those reference conditions.

3) Enhance downwash correction procedures to obtain correct transonic unsteady pressures based on steady reference conditions, independent of the reduced frequency of interest.

## II. Aerodynamic Model

Linear aerodynamic modeling techniques are based on discrete element solutions of the linear equations of the fluid flow. The fluid flow is represented by the linearized potential flow equation [25] in the frequency domain as

$$\beta^2\varphi_{xx} + \varphi_{yy} + \varphi_{zz} - 2ikM_\infty^2\varphi_x + k^2M_\infty^2\varphi = 0 \tag{1}$$

The aerodynamic modeling of unsteady linear potential flows may be performed by discrete kernel function methods, which are based on integral solutions of the small-disturbance linearized potential flow equation. The integral solution is obtained by the application of Green's theorem to this equation [26] in terms of unsteady source and doublet singularity distributions over the body surface $S$ and its associated wake surfaces $W$. The aerodynamic model for a given body is then approximated by the summation of elementary integral solutions associated with each element (panels) that discretizes the body surface. These elementary integrals at each panel, as well as the aerodynamic interference of one panel onto others, lead to a linear system of equations relating the pressure coefficient differences to downwash. The assembly of the elementary integral solutions results in a matrix in which the elements represent the aerodynamic influence of the panels into the control points. Each integral relationship between the downwash at a receiving point $i$, and the pressures at a sending panel $j$ may be written as a system of equations as

$$\varphi_z^i = w_i = D_{ij}\Delta C_{pj} \tag{2}$$

where each of the matrix elements $D_{ij}$ is the result of the integration of the kernel function over the given $j$th lifting-surface element geometry [14]:

$$D_{ij} = -\frac{1}{8\pi}\int_{\bar{\xi}_{j-1}}^{\bar{\xi}_j}\int_{\bar{\eta}_{j-1}}^{\bar{\eta}_j}\left\{\lim_{z\to 0}\left[\frac{\partial}{\partial z}K_\psi(x_i-\bar{\xi}_n, y_i-\bar{\eta}_n, 0, M_\infty, k)\right]\right\}\mathrm{d}\bar{\xi}_n\mathrm{d}\bar{\eta}_n \tag{3}$$

The inverse of the matrix operator $D$ multiplied by the downwash vector yields the pressure distribution. In other words, the solution of the system of equations gives the doublet strength at each panel referred to a given downwash that is related to a displacement-mode shape. The resulting inverse matrix is named as the aerodynamic influence coefficient matrix AIC, which is a function of the reduced frequency, and is related to the pressure distribution by

$$\{\Delta C_p(ik)\} = [\mathrm{AIC}(ik)]\{w(ik)\} \tag{4}$$

One should recall that a simple harmonic motion is assumed, hence the dependence on $ik$. The coefficients of this matrix may be interpreted as rates of pressure variation due to a given displacement amplitude input associated with the boundary conditions. Then the determination of the pressure coefficient vector in Eq. (4) is performed from the known downwash, which is related to the amplitude of the pitch and plunge motion at each element. The substantial derivative of a given displacement mode is composed of a derivative of the normal direction displacement with respect to the main flow direction plus the associated velocity scaled by the undisturbed flow speed, which, in a small-disturbance sense, represents an angle of attack. Therefore, from the boundary conditions for those small perturbations, the relationship between the normal wash and a solid boundary displacement is rewritten as

$$\{w(x,y,0,ik)\} = \frac{\partial h(x,y,0)}{\partial x} + ikh(x,y,0) = [F(ik)]\{h(x,y,0)\} \tag{5}$$

The substantial derivative applied to a given modal displacement vector $\{h\}$, which appears in Eq. (5), is denoted by the matrix operator $[F(ik)]$. The resulting aerodynamic loading vector $\{L_a(ik)\}$ may be expressed as the multiplication of the pressures by an integration matrix $S$, which is constructed from the panel elements geometrical characteristics. The resulting final expression for the unsteady loading over the lifting surface is given by

$$\{L_a(ik)\} = q_\infty[S][\mathrm{AIC}(ik)][F(ik)]\{h\} \quad \text{with } [F(ik)](\cdot) = \left[\frac{\partial(\cdot)}{\partial x} + ik(\cdot)\right] \tag{6}$$

The subsonic discrete kernel function approach will be further employed as the unsteady aerodynamic theory for computation of unsteady pressures and loads for aeroelastic response and stability analysis.

## III. Pressure-Matching Correction Method

The chosen approach to be further investigated is the downwash correction for pressure matching [3], taking into account steady reference nonlinear pressures as reference conditions to compute a weighting operator $WT$. In this situation, the computation of the weighting operator should be based on quasi-steady pressure slopes, as performed by Luber and Schmid [12]. The choice of the pressure rates instead of their absolute values is made to have displacement-independent weighting functions:

$$\{\Delta C_p^{\mathrm{nl}}(ik=0)\} = [\mathrm{AIC}(ik=0)][WT(ik=0)]\{w(ik=0)\} \tag{7}$$

This method presents good robustness and preservation of the mean steady nonlinear flow because the pressures are fully restored in steady state conditions. However, it should be noted that an identified discrepancy is its failure to compute unsteady pressures due to the absence of nonlinear unsteady pressure information in the reference conditions [3]. As an alternative, the unsteady pressure matching is a way to include the nonlinear unsteady information in the reference conditions:

$$\{\Delta C_p^{\mathrm{nl}}(ik_r)\} = [\mathrm{AIC}(ik_r)][WT(ik_r)]\{\bar{w}(ik_r)\} \Rightarrow \underbrace{[\mathrm{AIC}(ik_r)]^{-1}\{\Delta C_p^{\mathrm{nl}}(ik_r)\}}_{\{\bar{w}(ik_r)\}^{\mathrm{nl}}} = [WT(ik_r)]\{\bar{w}(ik_r)\} \tag{8}$$

where $k_r$ is a given reduced frequency associated with the unsteady transonic flow taken as reference condition, and $\{\bar{w}(ik_r)\}^{\mathrm{nl}}$ is the modified downwash resulting from the product of the AIC matrix and the reference pressures.

The computation of the downwash weighting matrices is performed from the ratio between the prescribed and the modified downwash for both methods using steady or unsteady pressures as reference conditions

$$[WT(ik=0)]_{ii} = \frac{\{w(ik=0)\}_i^{\mathrm{nl}}}{\{w(ik=0)\}_i} \qquad [WT(ik_r)]_{ii} = \frac{\{\bar{w}(ik_r)\}_i^{\mathrm{nl}}}{\{\bar{w}(ik_r)\}_i} \tag{9}$$

leading to diagonal weighting operators to be included in the following relation:

$$\{L_a^{\mathrm{nl}}(ik)\} = q_\infty[S][\mathrm{AIC}(ik)][WT(ik=0)]\{w(ik)\} = q_\infty[S][\mathrm{AIC}(ik)][WT(ik_r)]\{w(ik)\} \tag{10}$$

## IV. Pressure-Matching Versus Force-Matching Results

Giesing et al.'s [5] force-matching method was evaluated with respect to its performance in transonic aeroelastic stability analysis. This is a force-matching method in which the computation of the weighting matrix is based on strip loads. Each strip is composed of a set of panels along the streamwise direction. Because the reference loads are taken for each strip, whereas correction factors are to be calculated for each panel, it is clear that there are fewer available data (reference loads) than unknowns (correction factors). Therefore, the correction factors need to be computed by a minimization technique, which is described by Giesing et al.

The procedure was tested for the well-known AGARD wing 445.6 weakened model 3 with a standard aeroelastic configuration [27]. This wing does not present significant nonlinear effects under transonic flow conditions, because the flutter speed dip is mainly governed by compressibility effects. However, as pointed out by Bennet et al. [28], the AGARD 445.6 wing is a well-defined benchmark model for validation purposes, even for the investigation of transonic flow phenomena. The wing is modeled by the ZONA 6 method [26], implemented in the ZAERO software system [29], as an isolated wing attached to a wind-tunnel wall. The flow conditions that were used for analysis belong to a subset of the experiments on the wing aeroelastic stability presented by Yates [27]. In the same reference, the mathematical model regarding its structural and geometrical characteristics is described. The Mach numbers and air densities for the cases considered here are presented in Table 1.

The discrete element aerodynamic model, based on the ZONA 6 method, is composed of 220 panels uniformly distributed along the chord length. The flutter computations were performed using the $g$ method [29,30]. The stripwise lifting-surface loading was computed based on the integration of nonlinear steady pressure distributions, computed by the CFL3D Navier–Stokes computational fluid dynamics solver [31,32], The nonlinear flow simulations were performed at two distinct angles of attack of 0.0 and 0.5 deg. Quasi-steady pressure disturbances were computed and scaled by the amplitude of the motion, turning them independent of the amplitude of the disturbances, if one considers that the flow behaves linearly in this range.

Once these loads are obtained, the procedure allows the computation of a force-matching weighting matrix that exactly restores the nonlinear reference loading condition. In Fig. 1 the

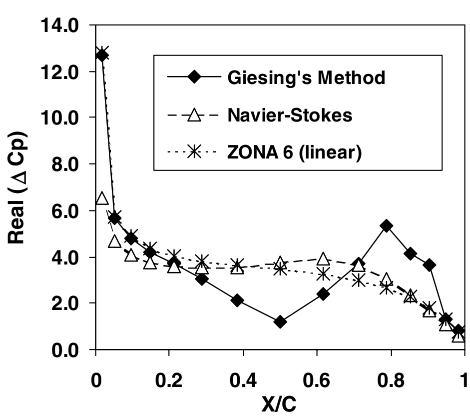
**Table 1 Flow conditions for AGARD wing 445.6 (weakened model 3) aeroelastic analysis**

| Mach | Reynolds | Density, slug/ft$^3$ |
|---|---|---|
| 0.678 | $1.410 \times 10^6$ | 0,000404 |
| 0.901 | $0.911 \times 10^6$ | 0,000193 |
| 0.960 | $0.627 \times 10^6$ | 0,000123 |

complex pressure distribution, after the multiplication of the weighting matrix from Giesing et al.'s [5] method, is compared with the nonlinear pressures obtained by the Navier–Stokes solution and with those obtained from the ZONA 6 linear solution [29].

The results from the flutter solution using Giesing et al.'s [5] correction procedure are presented in Fig. 2. In the same figure are results for the downwash weighting method (DWM) based on the same reference flow conditions used for Giesing et al.'s [5] method (i.e., for the same quasi-steady pressure disturbances).

Looking at Fig. 1, one can notice that the pressures are not restored because the force matching is a least-squares-based procedure; that is, there are less unknowns (which are the correction factors) than equations, because the weighting matrix needs to have the same dimension of the AIC matrix. Then this weighting matrix does not guarantee a correct pressure distribution, as it only assures the load matching. In Fig. 2, one should observe that the flutter stability margin is overpredicted in terms of the flutter dynamic pressure. This is an indication that such procedures have inadequately handled unsteady effects due to transonic nonlinearity, hence causing various discrepancies in their unsteady transonic pressure prediction, as shown in Fig. 1.

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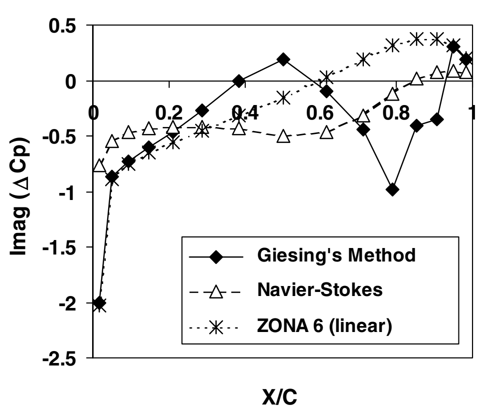


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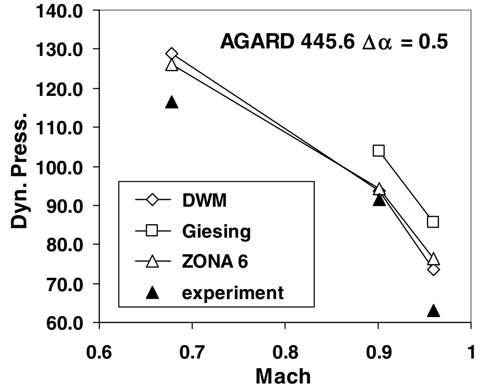
**Fig. 1 Comparison between the unsteady pressures obtained from Giesing et al.'s [5] method; ZONA 6 and CFL3D AGARD 445.6 wing; weakened model 3; $M_\infty = 0.96$.**

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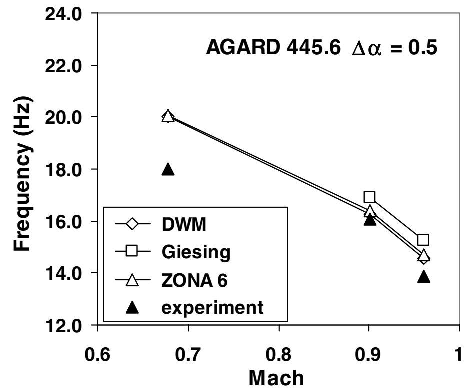


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**Fig. 2 Flutter boundaries (dynamic pressure and frequency) for the AGARD wing 445.6; weakened model 3 (Giesing et al. [5]).**

## V. Development of a New DWM

The following step is to develop a new downwash correction method beginning with the study on the transonic flow behavior in a small-disturbance context to understand the physics of transonic flows, including any linearity assumption [33,34]. In addition, an evaluation of downwash weighting methods using both steady and unsteady pressures as reference conditions is performed to understand the role of the reference conditions in the aeroelastic stability computations [3]. And finally, an enhancement of the downwash weighting procedure is introduced based on the observations from the aforementioned investigations [3].

The unsteady transonic flow behavior investigation [33] is based on an extension of Dowell et al. [35] research on this class of flows. However, an extension of this investigation is presented in the present effort because Dowell et al. employed a two-dimensional transonic small-disturbance (TSD) finite difference solution, whereas the present investigation is based on a three-dimensional Navier–Stokes finite difference computation. The features to be investigated are shock wave dynamics and loading ($C_L$ and $C_M$). The nonlinear aerodynamic model is based on the full Reynolds-averaged, 3-D Navier–Stokes equations. The numerical method is a second-order finite difference solution of the partial differential equations in structured grid using the Beam–Warming [36] approximate factorization scheme [37]. The results of the current Navier–Stokes implementation were previously validated with NLR F-5 wind-tunnel results [37,38].

The test case is the simulation of the same F-5 wing undergoing rigid pitch oscillations at different amplitudes of the motion and reduced frequencies. A Fourier transform is used to obtain frequency content of the time response of $C_p$, $C_L$, $C_{M1/4}$, and shock displacement. The latter quantity is defined as the location of the maximum pressure chordwise gradient [33]. The linear behavior of those quantities was observed with respect to the prescribed dynamic angles of attack leading to linear limits. Once these limits were identified, linear boundaries were constructed, as presented in Fig. 3. In this figure, $C_M$ represents the amplitude of the complex moment coefficient as a function of the dynamic angle of attack $\Delta\alpha$, and $k$ is a reduced frequency. A detailed description of this investigation is presented in [33].

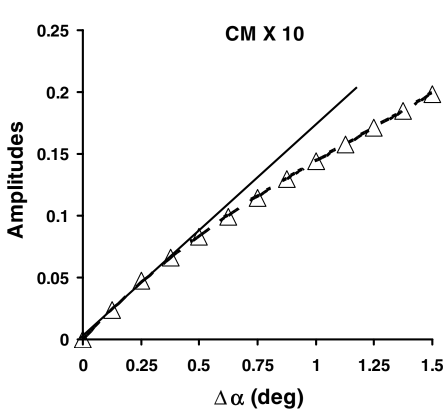
To identify the limit for each case, a linear equation was estimated so that it would start at the origin and pass through the first and second points, in which it is assumed that the flow behaves linearly at these low amplitudes of motion (0.125 and 0.250 deg). A linear equation was then reduced for these first values, leading to a linear relationship used to extrapolate to the following points. Thus, a percent deviation of the actual unsteady moment coefficient or shock displacement to the extrapolated value was computed. The first point at which the deviation exceeded 5% (named $C_{M\mathrm{crit}}$ for the case of the moment coefficient) and the previous point were used in an interpolation to find the corresponding dynamic angle of attack that defines the linear limit [33].

Looking at Fig. 3, it should be observed that linear limits depend on spanwise station, reduced frequency, and amplitude of the motion. This indicates that simply using two-dimensional linear limits, as presented by Dowell et al. [35], would not be appropriate when dealing with correction methods. Notwithstanding the dependence of the linear limits on reduced frequency and spanwise station, it is clear that some degree of linear behavior may be assumed, allowing the approximation of an unsteady transonic flow by means of linear small-disturbance governing equations.

The downwash weighting method is based on the matching of the pressures. Once the unsteady transonic flow behavior was characterized, the subsequent step was to perform an investigation regarding the role of the reference nonlinear pressures into the solution of the flutter problem of an aeroelastic system. The downwash weighting method was tested for the computation of the aeroelastic stability of the AGARD wing 445.6, weakened model 3 [27]. Computations were performed using either nonlinear steady [3] or unsteady pressure distribution as reference conditions [3]. The linear unsteady aerodynamic modeling is based on the doublet-lattice method, implemented in the MSC/NASTRAN software system [39]. The chosen flutter solution technique is the $pk$ method [40], which is mathematically consistent for the computation of the flutter boundary. The weighting operators are computed to correct the pressure to downwash relation, resulting from the modeling of the AGARD 445.6 wing using the doublet-lattice method.

The nonlinear pressures were obtained under unsteady motion to provide the corresponding reference conditions for the downwash weighting method. In this case, the pressures were computed under harmonic motion oscillations of the wing with amplitude $\Delta\alpha = 2.0$ deg, for which the nonlinear contribution due to unsteady transonic effects are more relevant. In the case of the application of the downwash correction method based on steady pressures as reference conditions [3], the chosen amplitude of the quasi-steady motion is $\Delta\alpha = 0.5$ deg because the nonlinear contribution comes from the steady mean transonic flow. The weighting operators are introduced in the aeroelastic analysis as correction factors, yielding the computed flutter speeds shown in Figs. 4 and 5. Figure 4 presents a comparison between the transonic flutter computation based on the correction using steady and unsteady nonlinear pressures and experimental results [27]. Figure 5 includes comparisons with some well-known aeroelastic analysis codes [28,29] with unsteady downwash correction procedure results.

The ZTAIC code [22,29] is a modal aerodynamic influence coefficient matrix correction method based on the transonic equivalent strip method developed by Liu et al. [22] and further extended by Chen et al. [41]. The CAP-TSD code is a time-domain finite difference solution of the transonic small-disturbance equation [28] coupled with a finite element structural dynamic model. The results shown in Figs. 4 and 5 are presented in terms of the flutter

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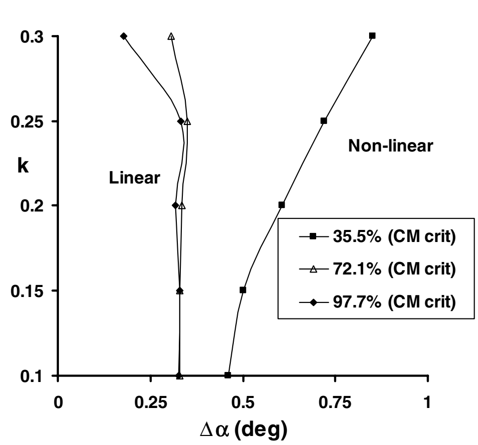


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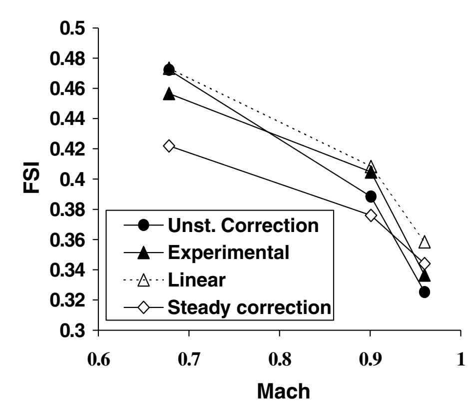
**Fig. 3 Observations on linear/nonlinear behavior.**

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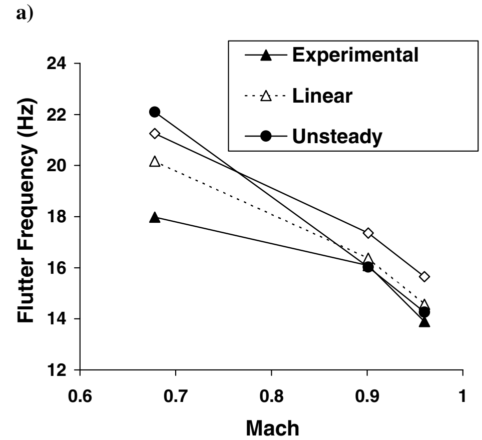


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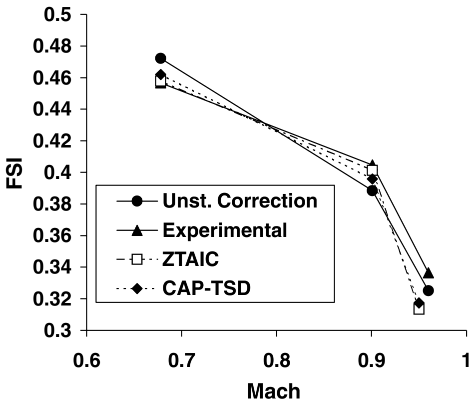
**Fig. 4 AGARD wing 445.6 results-comparison of the flutter computation results between steady and unsteady downwash weighting methods.**

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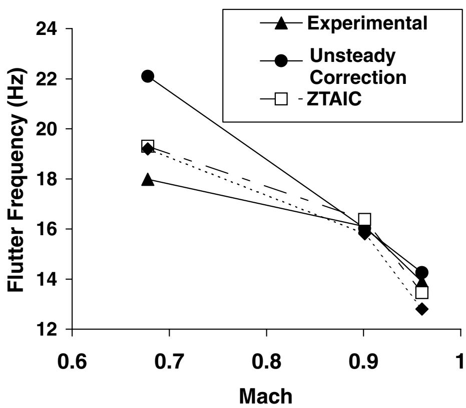


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**Fig. 5 AGARD wing 445.6 results-comparison of the flutter computation results between the unsteady downwash weighting methods and other methods.**

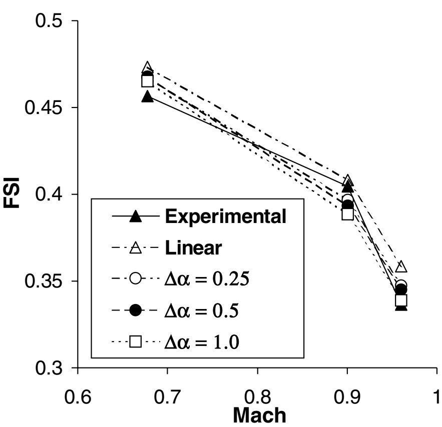
speed index (FSI) [27] and flutter frequency as a function of the Mach number. One can observe in Fig. 4 that the computed flutter speeds from the DWM based on unsteady pressures indicate the presence of the transonic dip. In the same figure, results are also shown from the correction based on steady pressures. One may notice that there is a good agreement when comparing the dip slope between the unsteady pressures based correction procedure and the experimental data. The steady pressure-based correction underestimates some of the flutter speeds as well as the dip slope. The reason for these discrepancies is related to the absence of a nonlinear unsteady pressure contribution, because the reference pressures, over which the correction factors are computed, are steady.

One feature to be pointed out in Fig. 4 is that in the subsonic Mach number case ($M_\infty = 0.678$), the correction based on unsteady pressures did not significantly change the linear prediction. This is because at the subsonic flow condition the differences between the amplitude of the linear and the nonlinear computed pressures are very small and not sufficient to introduce changes in the computation of the corrected flutter speed. However, the same behavior is not observed in the frequency plots shown in Fig. 4. Note that at $M_\infty = 0.678$ the flutter frequency computed from the unsteady downwash weighting method is higher than the linear result and also higher than that computed by the steady downwash correction method. The reason for this difference is not clear, but as may be seen in Fig. 5, there seems to be a tendency for overprediction of the flutter frequency in this subsonic case, even for other nonlinear codes such as CAP-TSD and ZTAIC, which better approach the experimental flutter frequencies.

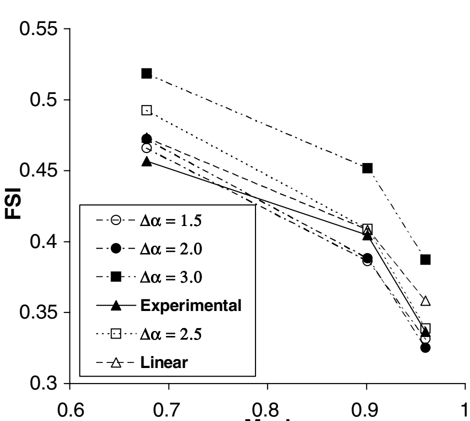
On the other hand, in the case of the computed flutter frequencies at $M_\infty = 0.901$ and 0.96, the frequencies are subjected to noticeable changes resulting in corrected values that are in better agreement with the experimental values. In Fig. 5, the CAP-TSD code calculations [28] also yield good results, because its formulation is based on a finite difference nonlinear solution of the three-dimensional form of the transonic small-disturbance equations. Furthermore, the ZTAIC and CAP-TSD codes adequately represent the severity in the flutter dip phenomenon, which is a desirable feature in transonic flutter prediction in this case. A disadvantage regarding such procedures is the dependency on two- and three-dimensional unsteady finite difference solutions of the nonlinear equations, respectively, increasing the computational cost in comparison with the downwash correction method.

The next step was to perform a sensitivity analysis with regard to the variation of the dynamic amplitude input, which is used to generate the nonlinear unsteady pressure distribution, taken as reference conditions for the computation of the correction factors. The objective is to understand the sensitivity of the computed aeroelastic system stability margins with respect to the amplitude of the motion responsible for generating the reference pressures. Figure 6 presents the computed flutter speeds, based on the unsteady downwash correction method, using the resulting pressures with respect to a set of displacement amplitudes. The same figure includes comparisons with the results computed from the uncorrected aeroelastic model and experimental measurements [27]. One should observe that the flutter speeds present significant variation with the nature of the unsteady pressure data used to compute the correction factors. These results are graphically represented in Fig. 6 as the variation of the nondimensional flutter speed (FSI) with the freestream Mach number.

The best results in approaching the experimental transonic-dip slope occur when one considers a dynamic angle amplitude of $\Delta\alpha = 2.0$ deg at the same time as the flutter speeds are slightly




a)




b)

**Fig. 6 Comparison between the flutter speed plots for several amplitudes of the motion.**

underestimated. Above this value, an interesting result should be noted. The computed flutter speeds at 2.5 deg are nearly coincident with the experimental speeds in the transonic Mach number range. Otherwise, it is possible to note an increase of the flutter speeds computed at 3.0 deg.

The present investigation indicates that computed flutter speeds using the unsteady downwash correction method depend on the amplitudes of the motion. In the linear/nonlinear investigations presented by Silva et al. [33], the transonic flow linear behavior limit could be established for disturbances in angle of attack below 0.35 deg, assuming the moment coefficient criterion. One should recall that moments play an important role in the flutter phenomenon. Hence, the variation of the flutter speeds and frequencies as a function of motion amplitude results from such nonlinear flow behavior of unsteady transonic flows. This result is consistent with the conclusions drawn by Silva et al. [33]. It is important to compute unsteady pressures inside the linear range; otherwise, spurious flutter predictions may appear. Further discussion on the sensitivity of the flutter speed with respect to the amplitude of the dynamic angle of attack was presented in [42].

The enhancement of the downwash weighting procedure is motivated by the aforementioned investigation results. Linear unsteady aerodynamic theories applied to the aeroelastic modeling and analysis are developed considering small disturbances around mean angle-of-attack variations of the lifting surface. The linear/nonlinear behavior investigation indicated that unsteady transonic flow behavior is strongly dependent on the amplitude of the motion [33]. However, aeroelastic deformations are usually smaller than the computed linear limits. Based on this observation, it is inferred that for aeroelastic analysis in a small-disturbance context, unsteadiness of transonic flows present a linear behavior with regard to aerodynamic derivatives and shock dynamics when the amplitudes of lifting surfaces undergoing unsteady motion are below the linear limits.

After examining the linear and nonlinear behavior of aerodynamic quantities as a function of the angle of attack, it was concluded that unsteady transonic pressures behaves linearly around a steady nonlinear mean flow for small amplitudes of the motion, such as aeroelastic deformations [33]. Therefore, the unsteadiness of transonic flow around a mean steady state nonlinear pressure distribution can be computed as a linear contribution predicted by a small-disturbance linear aerodynamic model in superposition to a nonlinear steady mean flow. Thus, the proposed procedure should be understood as an extension of the steady DWM, and the reference pressure distribution is composed of the superposition of an unsteady contribution predicted by the linearized potential flow equation in a steady nonlinear mean flow [3].

The procedure is divided in two steps, the first being a nonlinear steady mean flow correction as is performed for the correction of the steady downwash when nonlinear pressure differences are considered as reference conditions. The second step is the correction of the unsteady downwash, in which the unsteady counterpart of the reference pressures to be added to the steady nonlinear reference pressures will compose new reference pressure differences. These unsteady pressure contributions are predicted by a linear unsteady flow aerodynamic model.

In essence, the method employs a successive kernel expansion algorithm that can inject the given steady pressure into the perturbed-frequency-based integral equations to recover the out-of-phase pressure. To show the idea behind the successive kernel expansion method (SKEM), consider the lifting-surface formulation in which the integral equation according to the acceleration potential equation reads

$$w(x, y, 0, ik) = -\frac{1}{8\pi}\iint_A \Delta Cp(\xi, \eta) K(x-\xi, y-\eta, 0, ik, M)\mathrm{d}\xi \mathrm{d}\eta \tag{11}$$

where $K(ik)$ is the kernel function of the acceleration potential, $w(ik) = h_x + ikh$, and $h$ is the normal mode. It is assumed that it is possible to expand the subsonic kernel function as an asymptotic series around small reduced frequencies. The kernel $K(ik)$ is a function of reduced frequency and its domain of dependency is continuous and analytic. Consequently [43], the kernel function can be expanded in an asymptotic series for small reduced frequencies bounded by 1.0. Such previous attempts can be found in [44,45]. Therefore, with the expanded kernel, the pressure-downwash relation equation (11) can be recast in terms of a reduced frequency expansion format [43,46]. Expanding $\Delta C_p(ik)$ and $K(ik)$ in terms of $(ik)^n$ of Eq. (11) gives

$$\begin{aligned} h_x + ikh = (ik)^0 \iint_A (\Delta C_p^0 + ik\Delta C_p^1 + (ik)^2 \Delta C_p^2 + \cdots) \\ \times (K^0 + ikK^1 + (ik)^2 K^2 + \cdots)\mathrm{d}A \end{aligned} \tag{12}$$

Collecting like-order terms of $(ik)^n$ $(n = 0, 1, 2, \ldots)$ of Eq. (12) results in $n$ equations:

$$\begin{aligned} &\mathcal{O}(ik)^0\colon \{h_x\} = [K^0]\{\Delta C_p\} \rightarrow \Delta C_p^0 = [A^0]\{h_x\} \\ &\mathcal{O}(ik)^1\colon \{h\} = [K^1]\{\Delta C_p^0\} + [K^0]\{\Delta C_p^1\} \\ &\quad \rightarrow \Delta C_p^1 = [A^0]\{\{h_x\} - [K^1]\{\Delta C_p^0\}\} \\ &\mathcal{O}(ik)^2\colon \{0\} = [K^0]\{\Delta C_p^2\} + [K^1]\{\Delta C_p^1\} + [K^2]\{\Delta C_p^0\} \\ &\quad \rightarrow \Delta C_p^2 = [A^0]\{[K^1]\{\Delta C_p^1\} + [K^2]\{\Delta C_p^0\}\} \\ &\vdots \end{aligned} \tag{13}$$

where $[A^0] = [AIC(ik = 0)]$ is the AIC matrix at $k = 0$. Note that an additional term of order $k^2 \cdot \ln(k)$, between the order of $(ik)^1$ and $(ik)^2$, should have been formally presented in the expansion procedure [i.e., Eqs. (12) and (13)] [44,45]. This term has been omitted here, simply for clarity in describing the successive kernel expansion procedure.

The successive kernel expansion procedure injects the given steady pressure into Eq. (13) by replacing $A^0$ with AIC*, where AIC* is the corrected AIC matrix at $k = 0$ and it can be obtained through the downwash weighting matrix method [Eq. (8)]. The unsteady pressure is then computed using Eq. (11) for $\Delta C_p^n$ in a successive manner. In this way, a rational basis is established for the new DWM according to the successive kernel expansion procedure, whereby proper and consistent unsteady aerodynamic solutions can be obtained through given transonic steady aerodynamic inputs.

Because the kernel functions and their expansion attempts are given in [44,45], extension of the present new DWM approach for sonic and upper-transonic (where $M > 1.0$) unsteady flows are feasible.

## VI. Results and Discussions

### A. Validation of the Unsteady Pressure Distribution

Four test cases are selected to validate the unsteady pressure coefficients computed by ZTAW with measured data. These are an F-5 wing pitching about 50% root chord at $M = 0.948$ and $k = 0.264$ [38], a LANN wing in pitch mode about 62% root chord at $M = 0.822$ and $k = 0.105$ [47], and a Lessing wing in first bending mode at $M = 0.9$ and $k = 0.13$ [48].

The corrected AIC matrices at $k = 0$ are first generated using the downwash weighting matrix method. The unsteady pressure coefficients are then computed by the successive kernel expansion method. For all test cases, the CFL3D Navier–Stokes solver is used to compute the steady pressure coefficient $C_p$ at two angles of attack: $\alpha_1$ and $\alpha_2$. Therefore, the reference quasi-steady pressures, named here as $C_{p_{\mathrm{given}}}$, are obtained as the ratio between pressure coefficient differences and the amplitude of the motion.

### B. F-5 Wing Pitching about 50% Root Chord at $M = 0.948$ and $k = 0.264$

The CFL3D Navier–Stokes computations are performed at $M_\infty = 0.948$, $\alpha_1 = 0.5$ deg, and $\alpha = 0$ deg. Figure 7 shows that the steady pressure coefficient $C_p$ computed by CFL3D at this Mach number and $\alpha = 0.0$ deg correlates very well with the test data; $C_{p_{\text{given}}}$ is then computed accordingly,

$$\frac{C_p\ (\alpha = 0.5\,\text{deg}) - C_p\ (\alpha = 0.0\,\text{deg})}{0.5\,\text{deg}}$$

and it is presented in Fig. 8.

Shown in Fig. 9 is the comparison of the unsteady $C_p$ computed by ZTAW and the DWM method (due to the pitch oscillation about 50% root chord at $M_\infty = 0.948$ and $k = 0.264$) with the wind-tunnel measured data. It can be seen that the real parts of the unsteady pressures $\Delta C_p$ computed by ZTAW and DWM are very close to $C_{p_{\text{given}}}$ and they agree well with the experiments [38]. As discussed previously, this is expected because the in-phase $\Delta C_p$ of ZTAW and that of DWM can be essentially derived from $C_{p_{\text{given}}}$. However, the imaginary parts of the unsteady pressures $\Delta C_p$ computed by DWM do not seem to include the shock-jump behavior as indicated by the measured data. By contrast, the correct shock-jump behavior is well predicted by ZTAW. This case clearly shows the shortcoming of the DWM method and the ability of the ZTAW method to extract accurate out-of-phase pressures from the given steady $C_p$ through the SKEM.

### C. LANN Wing in Pitch Mode about 62% Root Chord at $M = 0.822$ and $k = 0.105$

The CFL3D steady $C_p$ presented in Fig. 10 on a LANN wing at $M_\infty = 0.822$ and $\alpha = 0.6$ deg shows a strong shock located at 40% chord. $C_{p_{\text{given}}}$ for this case is depicted in Fig. 11 and was computed by CFL3D at $\alpha = 0.6$ and 0.8 deg according to

$$C_{p_{\text{given}}} = \frac{\Delta C_p}{\Delta \alpha} = \frac{C_p\ (\alpha = 0.8\,\text{deg}) - C_p\ (\alpha = 0.6\,\text{deg})}{0.2\,\text{deg}}$$

The unsteady pressures $\Delta C_p$ on the LANN wing in pitch oscillation about 62% root chord at $M_\infty = 0.822$ and $k = 0.105$ computed by ZTAW and DWM are presented in Fig. 12. This time it is seen that the imaginary parts of the unsteady pressures $\Delta C_p$ as computed by the DWM method result in erroneous shock-jump behavior of an opposite trend to that of the measured data.

Again, in contrast, the computed results of unsteady $\Delta C_p$ by ZTAW predicted the correct trend in shock-jump behavior and are in good agreement with measured data [47] throughout all spanwise locations. It should be noted that the zigzag behavior of the ZTAW unsteady $C_p$ at $y/2b = 82.5\%$ is caused by the same zigzag behavior of $C_{p_{\text{given}}}$ at the same spanwise station. If more accurate $C_{p_{\text{given}}}$ is provided by CFL3D, it is believed that more accurate unsteady $\Delta C_p$ is expected to be obtained by ZTAW.

### D. Lessing Wing in First Bending Mode at $M = 0.9$ and $k = 0.13$

An important issue exists for almost all transonic AIC correction methods, which is their capability of generating accurate real and imaginary parts of unsteady pressures $\Delta C_p$ of elastic modes with given steady pressure and the $C_p$ given based on a static pitch mode. The Lessing wing [48] is an ideal case to clarify this issue because the measured unsteady $\Delta C_p$ on the Lessing wing is obtained by an oscillating first bending mode, not a pitch mode. Shown in Figs. 13 and 14 are the steady $C_p$ and $C_{p_{\text{given}}}$, respectively, computed by CFL3D at $M = 0.9$. It should be noted that $C_{p_{\text{given}}}$ is obtained here by Eq. (8), which is due to a pitch mode at $k = 0$.

The unsteady $C_p$ due to an oscillating first bending mode at $M = 0.9$ and $k = 0.13$ is presented in Fig. 15, in which excellent agreement between the SKEM results and wind-tunnel data can be seen. This agreement clearly assures the applicability of ZTAW to provide proper unsteady pressures of elastic modes with given static pitch-mode solutions. Meanwhile, the results computed by the linear method (ZONA 6) shown in Fig. 15 fails to predict the unsteady shock effects, as expected.

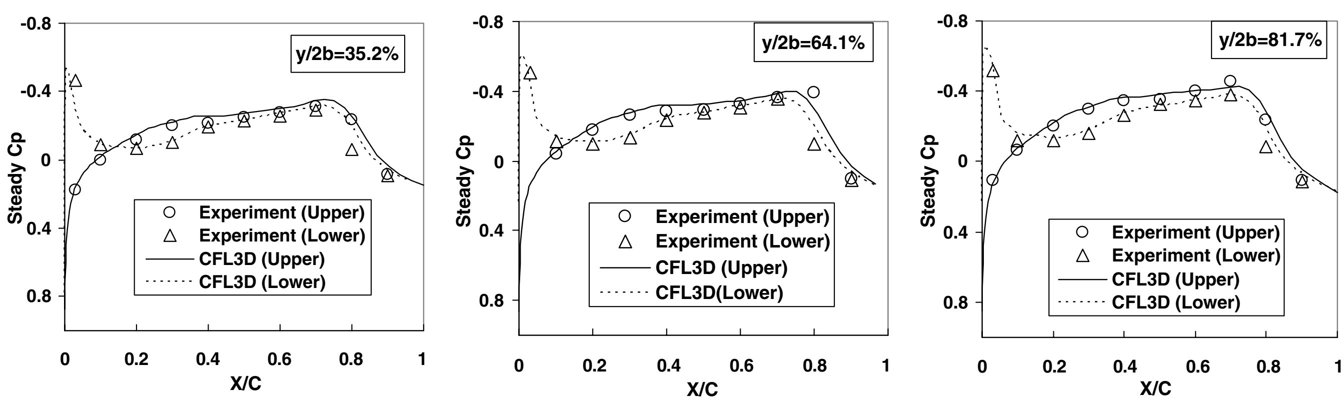



**Fig. 7 Comparison of steady $C_p$ between CFL3D and wind-tunnel data on a F-5 wing at $M_\infty = 0.948$ and $\alpha = 0.0$ deg.**

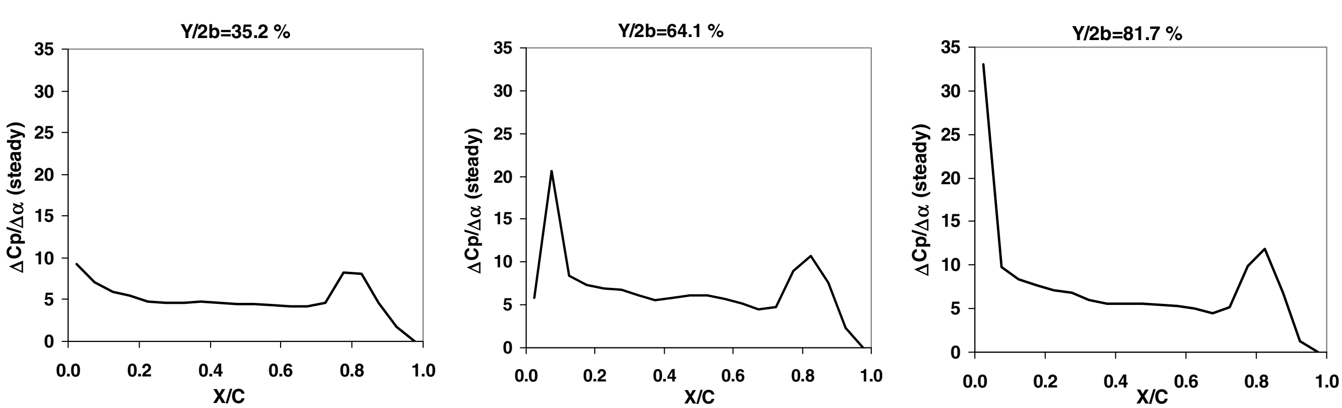



**Fig. 8 Pressure coefficient ratio for the F-5 wing at $M_\infty = 0.948$ and $\Delta\alpha = 0.5$ deg ($C_{p_{\text{given}}}$).**

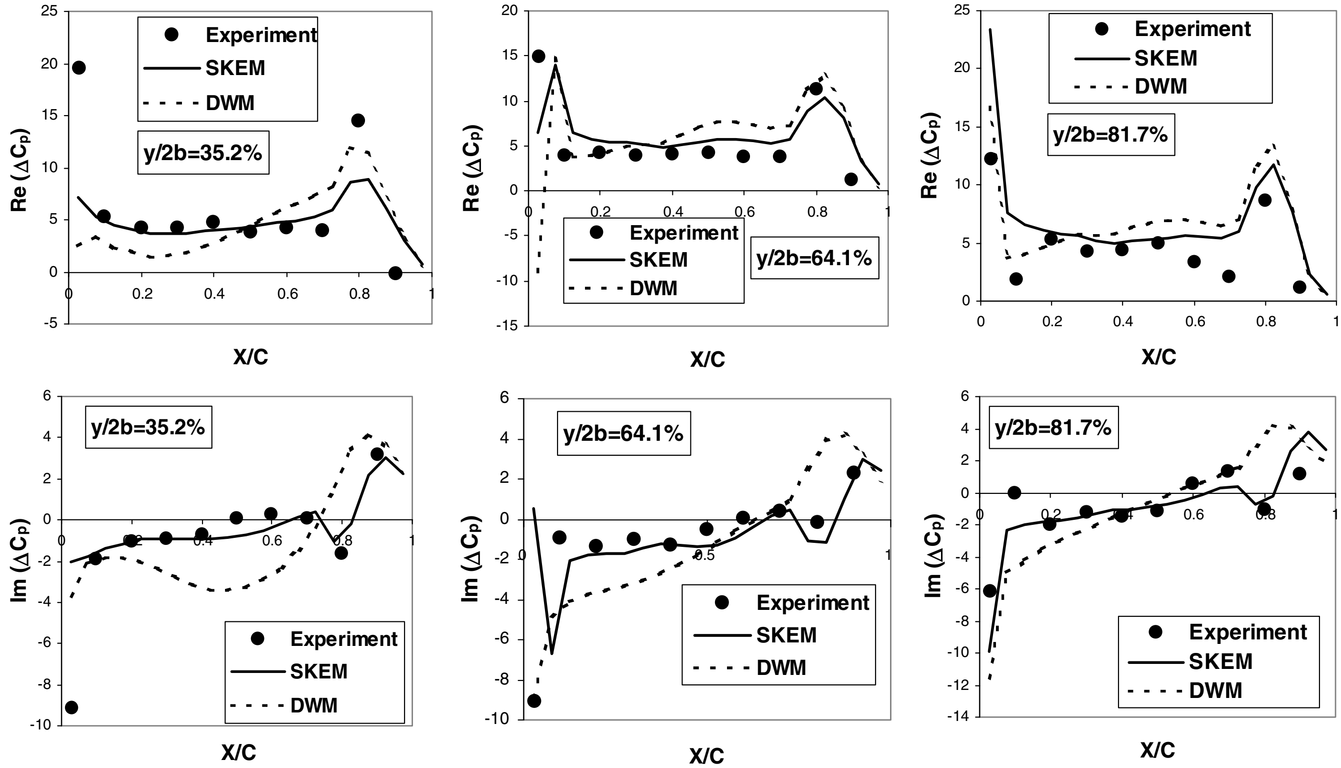



Fig. 9 Unsteady $\Delta C_p$ on a F-5 wing due to a pitch oscillation about 50% chord at $M_\infty = 0.948$ and $k = 0.264$.

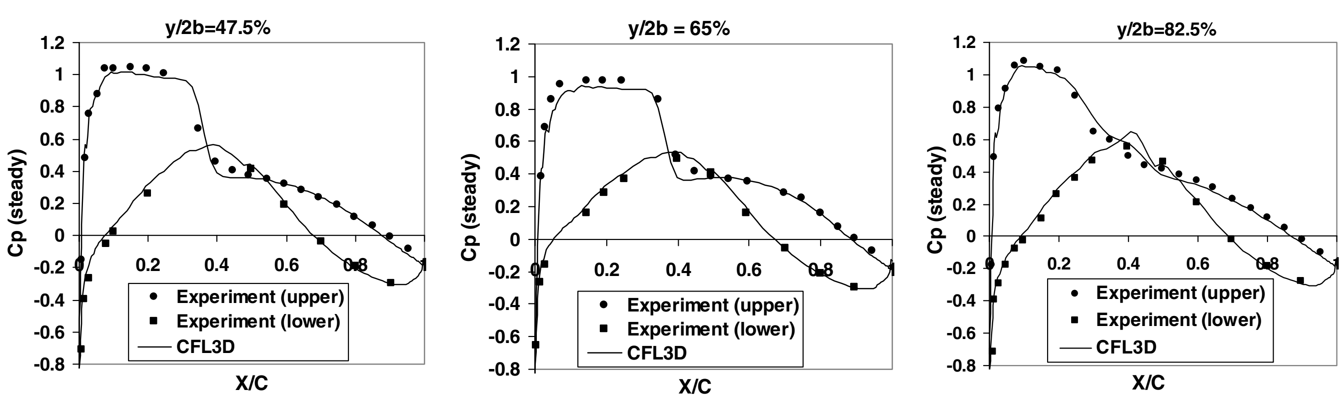



Fig. 10 Steady $C_p$ on a LANN wing at $M_\infty = 0.822$ and $\alpha = 0.6$ deg.

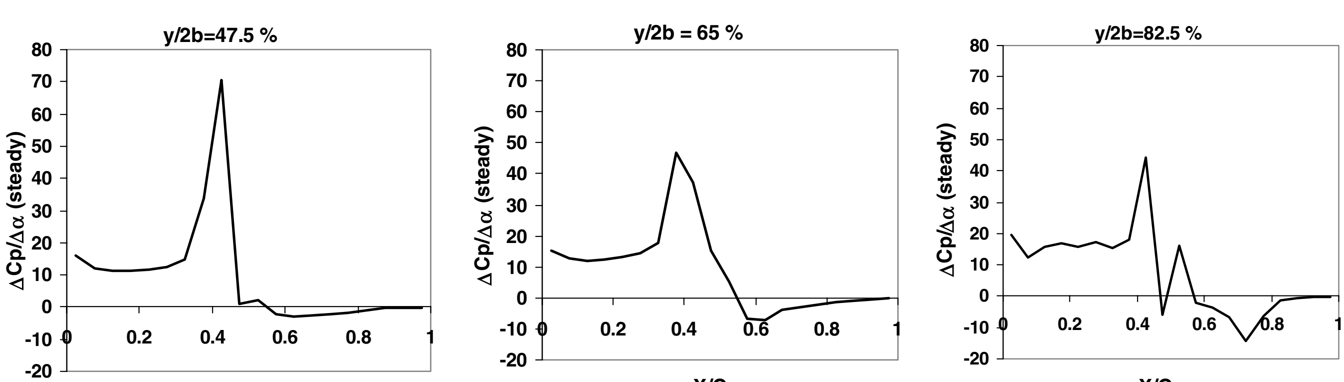



Fig. 11 $C_{p_{\mathrm{given}}}$ Computed by CFL3D of a LANN wing at $M_\infty = 0.822$.

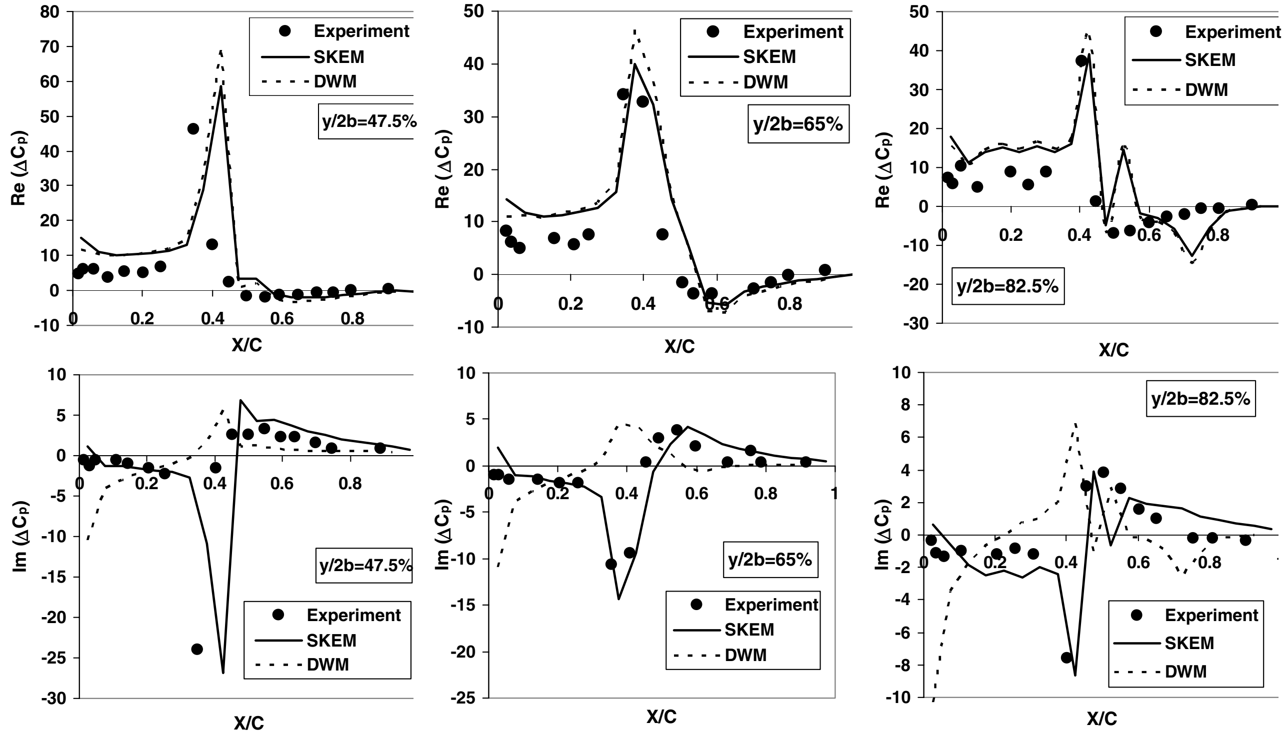



Fig. 12 Unsteady $\Delta C_p$ on a LANN wing due to a pitch oscillation about 62% root chord at $M_\infty = 0.822$ and $k = 0.105$.

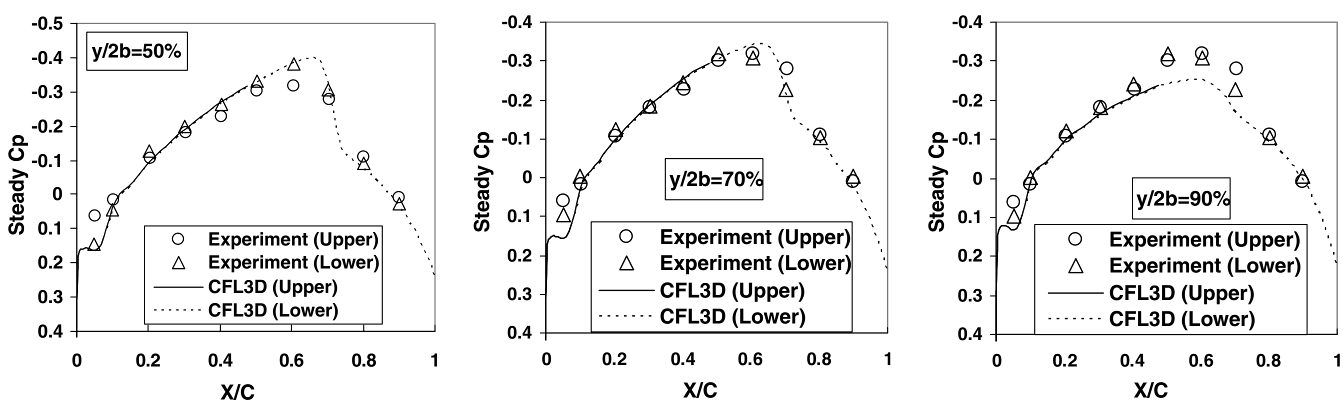



Fig. 13 Comparison of steady $C_p$ between the CFL3D and wind-tunnel results on a Lessing wing at $M_\infty = 0.9$ and $\alpha = 0.0$ deg.

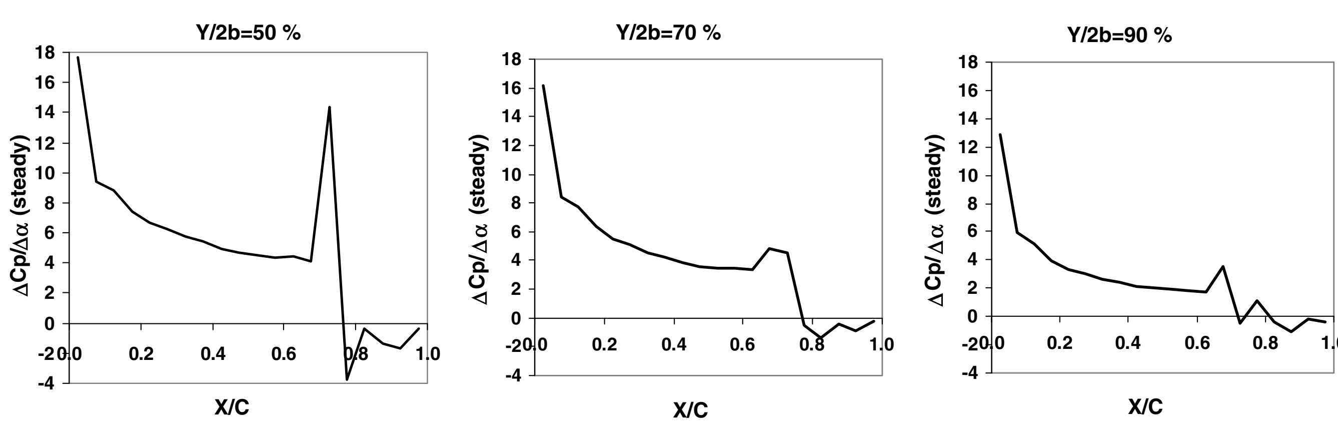



Fig. 14 $C_{p_{\text{given}}}$ computed by CFL3D of a Lessing wing at $M_\infty = 0.9$.

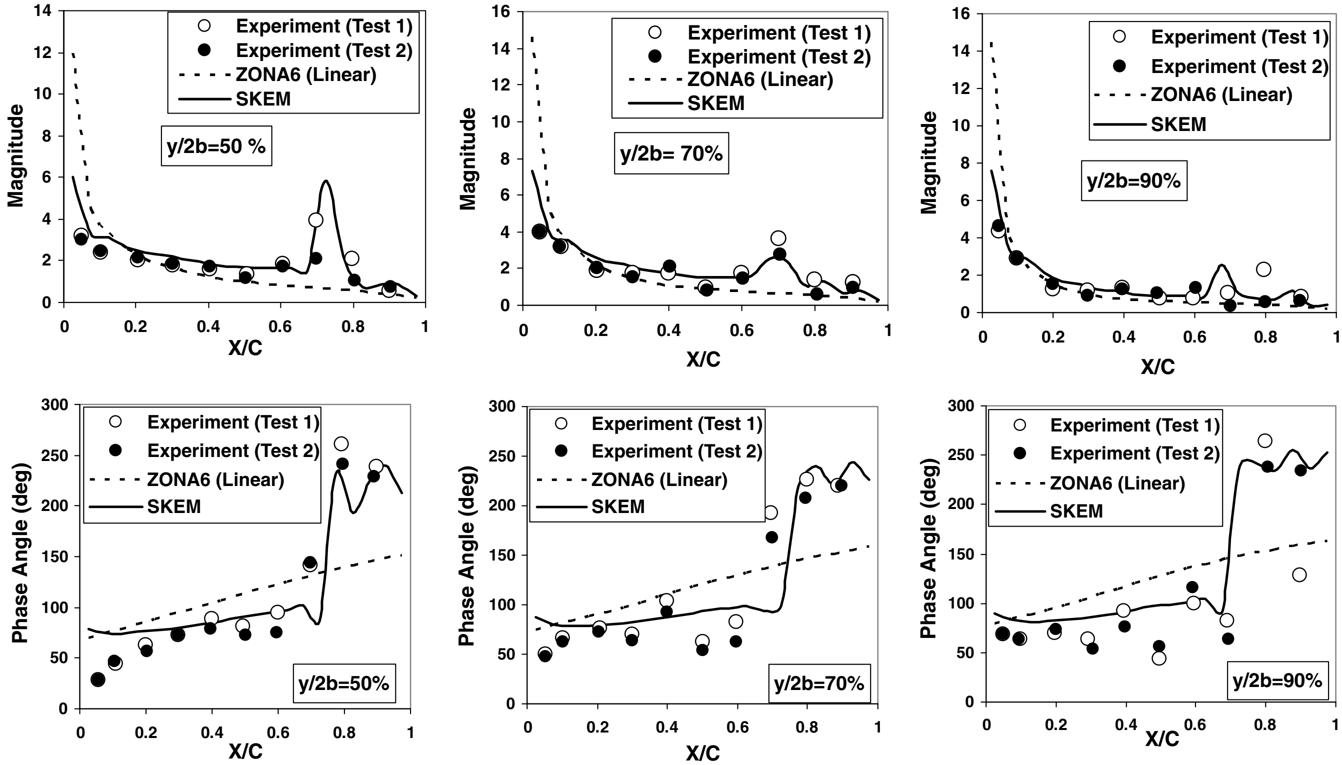



Fig. 15 Magnitude and phases angles of the unsteady pressures for the Lessing wing at $M_\infty = 0.9$ and reduced frequency $k = 0.13$.

### E. Validation of Flutter-Boundary Predictions

Two test cases are selected to validate the flutter-boundary predictions of ZTAW with the wind-tunnel measurements. Again, $C_{p_{\text{given}}}$ is obtained by CFL3D at two angles of attack and computed by Eq. (13).

### F. Flutter Boundaries of the AGARD 445.6 Wing

Four flutter boundaries of the AGARD 445.6 weakened wing [27] are presented in Fig. 16: those computed with ZTAW, DWM, and ZONA 6, as well as wind-tunnel measurements.

In the steady downwash correction the pressure phases are not corrected, resulting in more conservative flutter dynamic pressures when compared with strictly linear results. In addition, one should note that the results of this method for transonic Mach numbers have the same behavior of the uncorrected linear methods regarding the transonic-dip curve slope. This is an indication that the transonic-dip behavior is closely related to the contribution of the imaginary part of the pressures.

In the case of the successive kernel expansion procedure, the computed correction factors result from the matching of nonlinear unsteady pressures composed by nonlinear steady mean flow pressures and linear unsteady pressure contributions. Therefore, the correction factor will take into account (in an approximate form) the unsteady transonic flow behavior, because its computation is referred to those nonlinear unsteady pressures. In summary, the dip slope predicted by the successive kernel expansion method is increased, because the correction procedure takes into account the influence of the real and imaginary parts of the transonic unsteady pressures. Thus, this improvement in the reference conditions leads to better results in approaching the experimental measurements.

### G. Flutter Boundaries of the PAPA Wing

The flutter boundaries of the pitch-and-plunge apparatus (PAPA) wing [49] at $\alpha = 1$ and $-2$ deg computed by DWM and ZTAW (SKEM) are presented in Fig. 17 and compared with the wind-tunnel measurements. It can be seen that DWM largely underpredicts the flutter dynamic pressures, whereas they are well predicted by ZTAW. A similar trend of the flutter boundaries of the PAPA wing at $\alpha = 2$ deg is shown in the same figure. Again, ZTAW flutter results agree better with the test data than those of DWM. One should

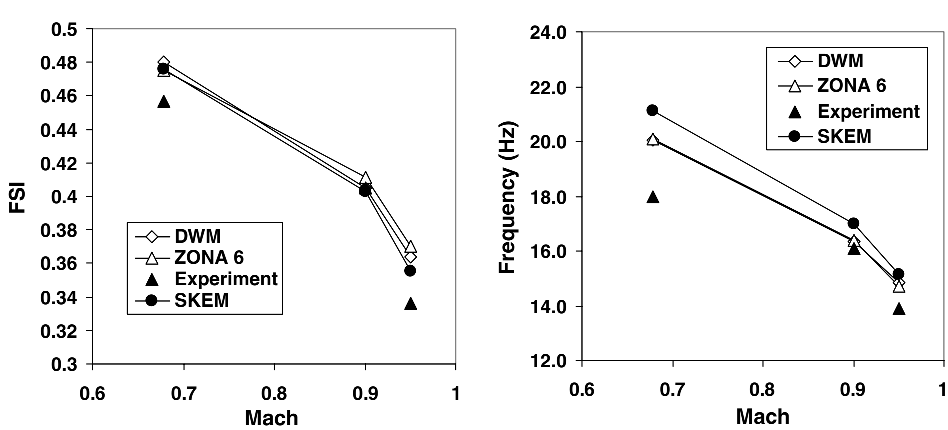



Fig. 16 Flutter boundary of the AGARD 445.6 wing weakened model 3 [27].

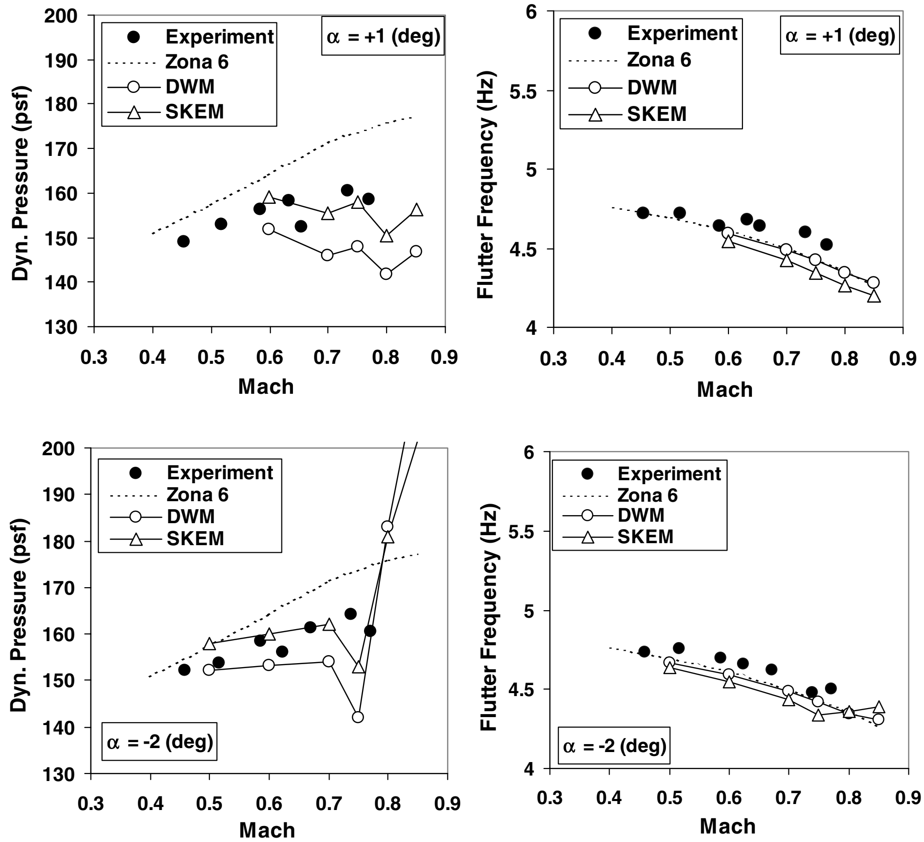


**Fig. 17 Flutter boundaries of the PAPA wing [49].**



observe that the downwash correction method largely underpredicts the flutter dynamic pressure, whereas the flutter results of the successive kernel expansion method correlate well with test data, both for the subsonic and the transonic Mach numbers. This is so because the unsteady components of the nonlinear pressures introduced by the correction procedure play an important role in the flutter computation.

Another feature to be highlighted with regard to the successive kernel expansion method is the capability to predict the flutter of the wing at different initial angles of attack, as opposed to the linear theory, which is independent of angle of attack. This is because the steady mean flow can be computed at a given steady state angle of attack, and the nonlinear unsteady pressures, used to compute the correction factors, will be obtained from the contribution of the nonlinear steady mean flow pressures and added to unsteady linear pressures. Therefore, the angle-of-attack contribution is included in the mean flow conditions, because unsteady linear pressures predicted by the linear unsteady aerodynamic model are independent of the angle of attack.

## VII. Conclusions

The transonic dips were captured with application of the present steady and unsteady downwash correction methods. However, the dip phenomenon was more evident and closer to experiments when the unsteady downwash correction method was employed. It may be concluded that the unsteady flow contribution, embedded in the CFD computed nonlinear unsteady pressures taken as reference conditions, plays an important role in the transonic flutter phenomenon. Another feature to be highlighted is that the flutter boundaries computed by unsteady downwash correction methods are significantly dependent on the unsteady pressures taken as reference conditions. The preceding is consistent with the observations regarding the linear/nonlinear investigations, in which it was found that the linear limit associated with the lifting moment is more sensitive to the amplitude of the motion used to compute the nonlinear unsteady pressures for the unsteady correction method [33,42].

Almost all previous AIC correction methods based on steady reference conditions are found to yield erroneous out-of-phase pressures, especially in terms of shock-jump behavior. The SKEM, also named the ZTAW method, employs a kernel expansion procedure that forms a rational basis for an advanced DWM method. With quasi-steady pressures computed from a rigid pitch displacement of the lifting surface given either by CFD or measurements, ZTAW can yield accurate out-of-phase pressures in a general frequency range as well as well-correlated flutter solutions.

In addition, ZTAW can be extended to cover the full transonic range, including $M \geq 1.0$, because the successive kernel expansion method is derived according to a unified acceleration potential formulation covering subsonic, sonic, and supersonic Mach numbers.

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## References

[1] Ashley, H., "Role of Shocks in the 'Sub-Transonic' Flutter Phenomenon," *Journal of Aircraft*, Vol. 17, No. 3, Mar. 1980, pp. 187–195.
doi:10.2514/3.57891

[2] Palacios, R., Climent, H., Karlsson, A., and Winzell, B., "Assessment of Strategies for Correcting Linear Unsteady Aerodynamics Using CFD or Test Results," *Proceedings of the CEAS/AIAA International Forum on Aeroelasticity and Structural Dynamics*, Confederation of European Aerospace Societies, Madrid, June 2001, pp. 195–210.

[3] Silva, R. G. A., "A Study on Correction Methods for Aeroelastic Analysis in Transonic Flow," Ph.D. Dissertation, Aeronautics Dept.,

Technological Inst. of Aeronautics, São José dos Campos, Brazil, 2004.

[4] Yates, E. C., Jr., "Modified-Strip Analysis Method for Predicting Wing Flutter at Subsonic to Hypersonic Speeds," *Journal of Aircraft*, Vol. 3, No. 1, Jan.–Feb. 1966, pp. 25–29.

[5] Giesing, J. P., Kalman, T. P., and Rodden, W. P., "Correction Factor Techniques for Improving Aerodynamic Prediction Methods," NASA CR-144967, 1976.

[6] Zwaan, R. J., "Verification of Calculation Methods for Unsteady Airloads in the Prediction of Transonic Flutter," *Journal of Aircraft*, Vol. 22, No. 10, 1985, pp. 833–839.

[7] Pitt, D. M., and Goodman, C. E., "Flutter Calculations Using Doublet Lattice Aerodynamics Modified by the Full Potential Equations," 28th AIAA/ ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference, AIAA Paper 87-0882-CP, Monterey, CA, 1987.

[8] Brink-Spalink, J., and Bruns, J. M., "Correction of Unsteady Aerodynamic Influence Coefficients Using Experimental or CFD Data," 41st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference and Exhibit, AIAA Paper 2000-1498, Atlanta, Apr. 2000.

[9] Rodden, W. P., and Revell, J. D., "Status of Unsteady Aerodynamic Influence Coefficients," Inst. of Aeronautical Sciences, Paper FF-33, 1962.

[10] Bergh, H., and Tijdeman, H., "Binary Flutter Calculations with Theoretical and Empirical Aerodynamic Derivatives for a Wing-Control Surface System in Two-Dimensional Incompressible Flow," National Aerospace Lab., Rept. NLR TR 68069 U, Amsterdam, 1968.

[11] Bergh, H., and Zwaan, R. J., "A Method for Estimating Unsteady Pressure Distributions for Arbitrary Vibration Modes from Theory and form Measured Distributions for One Single Mode," National Aerospace Lab., Rept. NLR-TR F.250, Amsterdam, 1966.

[12] Luber, W., and Schmid, H., "Flutter Investigations in the Transonic Flow Regime for a Fighter Type Aircraft," AGARD, Rept. RN 703, Paris, Sept. 1982.

[13] Baker, M. L., "CFD Based Corrections for Linear Aerodynamic Methods," AGARD, Rept. R822, Paris, Oct. 1997.

[14] Baker, M. L., Yuan, K., and Goggin, P. J., "Calculation of Corrections to Linear Aerodynamic Methods for Static and Dynamic Analysis and Design," 39th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit, AIAA Paper 98-2072, Long Beach, CA, Apr. 1998.

[15] Jadic, I., Hartley, D., Giri, J., "An Enhanced Correction Factor Technique for Aerodynamic Influence Coefficient Methods," *1999 MSC Aerospace User's Conference* [online proceedings], http://www.mscsoftware.com/support/library/conf/auc99/p02299.pdf [retrieved Oct. 2008].

[16] Jadic, I., Hartley, D., and Giri, J., "Improving the Linear Aerodynamic Approximation in Linear Aeroelasticity," In: 41st Structures, Structural Dynamics and Materials Conference and Exhibit, AIAA Paper 2000-1450, Atlanta, Apr. 2000.

[17] Garner, H. C., "A Practical Framework for Evaluation of Oscillating Aerodynamic Loadings on Wings in Supercritical Flows," *RAE Technical Memorandum Structures 900*, Royal Aircraft Establishment, London, 1977, pp. 16-1–16-15.

[18] Dau, K., "A Semi-Empirical Method for Calculating Pressures on Oscillating Wings in Unsteady Transonic Flow," Deutsche Airbus, Rept. DA-EF24-B08/92, Bremen, Germany, Oct. 1992.

[19] SenGupta, G., "Evaluation of the Dau-Garner Method for Predicting Unsteady Pressures in Transonic Flow," International Symposium on Aeronautical Science and Technology Paper 96-1.3.4, June 1996.

[20] Yonemoto, K., "A Practical Method Predicting Transonic Wing Flutter Phenomena," *14th International Council of the Aeronautical Sciences Congress (ICAS-1984)*, AIAA, New York, 1984, pp. 724–732.

[21] Suciu, E., Glaser, J., and Coll, R., "Aerodynamic Derivatives Factoring Scheme for the MSC/NASTRAN Doublet Lattice Program," *MSC/NASTRAN Worlds Users' Conference*, 1990, http://www.mscsoftware.com/support/library/conf/wuc90/p01890.pdf [retrieved Oct. 2008].

[22] Liu, D. D., Kao, Y. F., and Fung, K. Y., "An Efficient Method for Computing Unsteady Transonic Aerodynamics of Swept Wings with Control Surfaces," *Journal of Aircraft*, Vol. 25, No. 1, 1988, pp. 25–31. doi:10.2514/3.45536

[23] Chen, P. C., Sarhaddi, D., and Liu, D. D., "Transonic Aerodynamic Influence Coefficient Approach for Aeroelastic and MDO Applications," *Journal of Aircraft*, Vol. 37, No. 1, 2000, pp. 85–94. doi:10.2514/2.2565

[24] McCain, W. E., "Measured and Calculated Airloads on a Transport Wing Model," *Journal of Aircraft*, Vol. 22, No. 4, 1985, pp. 336–342. doi:10.2514/3.45129

[25] Landahl, M. T., *Unsteady Transonic Flow*, Pergamon, New York, 1951.

[26] Chen, P. C., Lee, H. W., and Liu, D. D., "Unsteady Subsonic Aerodynamics for Bodies and Wings with External Stores Including Wake Effect," *Journal of Aircraft*, Vol. 30, No. 5, Sept.–Oct. 1993, pp. 618–628. doi:10.2514/3.46390

[27] Yates, E. C., Jr., "AGARD Standard Aeroelastic Configurations for Dynamic Response I—Wing 445.6," AGARD, Rept. 765, Paris, 1988.

[28] Bennet, R. M., Batina, J. T., and Cunningham, H. J., "Wing Flutter Calculations with the CAP-TSD Unsteady Transonic Small-Disturbance Program," *Journal of Aircraft*, Vol. 26, No. 9, 1989, pp. 876–882. doi:10.2514/3.45854

[29] ZAERO, Software Package, Ver. 6.5, Zona Technology, Scottsdale, AZ, 2003.

[30] Chen, P. C., "Damping Perturbation Method for Flutter Solution: The *g* Method," *AIAA Journal*, Vol. 38, No. 9, 2000, pp. 1519–1524. doi:10.2514/2.1171

[31] Thomas, J. L., Krist, S. L., and Anderson, W. K., "Navier-Stokes Computations of Vortical Flows over Low-Aspect-Ratio Wings," *AIAA Journal*, Vol. 28, No. 2, 1990, pp. 205–212. doi:10.2514/3.10376

[32] Rumsey, C., Biedron, R., Thomas, J., "CFL3D: Its History and Some Recent Applications," *Godunov's Method for Gas Dynamics*, Dept. of Aerospace Engineering, Michigan Univ., Ann Arbor, MI, May 1997; also NASA TM 112861, 1997.

[33] Silva, R. G. A., Mello, O. A. F., and Azevedo, J. L. F., "A Navier-Stokes Based Study into Linearity in Transonic Flow for Flutter Analysis," *Journal of Aircraft*, Vol. 40, No. 5, 2003, pp. 997–1000. doi:10.2514/2.6886

[34] Silva, R. G. A., Mello, O. A. F., and Azevedo, J. L. F., "A Numerical Study into Shock Displacement Effects for Aeroelastic Analysis in Transonic Flow," *Proceedings of the 9th Brazilian Congress of Thermal Engineering and Sciences (ENCIT 2002)*, Associação Brasileira dos Criadores de Marchigiana, Caxambú, Brazil, Oct. 2002.

[35] Dowell, E. H., Bland, S. R., and Williams, M. H., "Linear/Nonlinear Behavior in Unsteady Transonic Aerodynamics," *AIAA Journal*, Vol. 21, No. 1, Jan. 1983, pp. 38–46. doi:10.2514/3.8025

[36] Beam, R. M., and Warming, R. F., "An Implicit Factored Scheme for the Compressible Navier-Stokes Equations," *AIAA Journal*, Vol. 16, Apr. 1978, pp. 393–402. doi:10.2514/3.60901

[37] Mello, O. A., F., "An Improved Hybrid Navier-Stokes/Full-Potential Method for Computation of Unsteady Compressible Viscous flows," Ph.D. Thesis, Georgia Inst. of Technology, Atlanta, Nov. 1994.

[38] Tijdeman, H., van Nunen, J. W. G., Kraan, A. N., Persoon, A. J., Poestkoke, R., Roos, R., Schippers, P., and Siebert, C. M., "Transonic Wind Tunnel Tests on an Oscillating Wing with External Stores, Part 2: Clean Wing," U.S. Air Force Flight Dynamics Lab., Rept. AFFDL-TR-78-194, Wright–Patterson AFB, OH, 1979; also National Aerospace Lab., Rept. NLR-TR-78106U, Pt. 2, Amsterdam.

[39] MSC/NASTRAN, Software Package, Ver. 68.2, MacNeal-Schwendler Corp., Los Angeles, 1995.

[40] Hassig, H. J., "An Approximate True Damping Solution of the Flutter Equation by Determinant Iteration," *Journal of Aircraft*, Vol. 8, No. 11, 1971, pp. 885–889. doi:10.2514/3.44311

[41] Chen, P. C., Sarhaddi, D., and Liu, D. D., "Transonic Aerodynamic Influence Coefficient Approach for Aeroelastic and MDO Applications," *Journal of Aircraft*, Vol. 37, No. 1, 2000, pp. 85–94. doi:10.2514/2.2565

[42] Silva, R. G. A., Mello, O. A. F., and Azevedo, J. L. F., "Sensitivity Study of Downwash Weighting Methods for Transonic Aeroelastic Stability Analysis," *Journal of Aircraft*, Vol. 43, No. 5, Sept.–Oct. 2006, pp. 1506–1515. doi:10.2514/1.16886

[43] Miles, J. W., Potential Theory of Unsteady Supersonic Flow, Cambridge Univ. Press, London, 1959.

[44] Watkins, C. E., Runyan, H. L., and Woolson, D. S., "On the Kernel Function of the Integral Equation Relating the Lift and Downwash Distributions of Oscillating Finite Wings in Subsonic Flow," NACA, Rept. 1234, 1955.

[45] Laschka, B., "Zur Theorie der Harmonisch Schwingenden Tragenden Flache bei Unterschallanstromung," *Zeitschrift fur Flugwissenschaften*, Vol. 11, No. 7, 1963, pp. 265–292.

[46] Liu, D. D., "A Mixed Kernel Function Method for Unsteady Transonic Flow Calculations," Northrop Corp., Aircraft Div. , Rept. PC20-59, Hawthorne, CA, Mar. 1976.

[47] Malone, J. B., and Ruo, S. Y., "LANN Wing Test Program: Acquisition

and Application of Unsteady Transonic Data for Evaluation of Three-Dimensional Computational Methods," U.S. Air Force Flight Dynamics Lab., Rept. AFWAL-TR-83-3006, Wright–Patterson, AFB, OH, Feb. 1983.

[48] Lessing, H. C., Troutman, J. L., and Menees, G. P., "Experimental Determination on a Rectangular Wing Oscillating in the First Bending Mode for Mach Numbers from 0.24 to 1.30," NASA Rept. TND-33, Dec. 1960.

[49] Farmer, M. G., and Rivera, J. A., "Experimental Flutter Results of a Cambered Supercritical Wing on a Pitch and Plunge Apparatus," *Aerospace Flutter and Dynamics Council Meeting*, NASA Langley Research Center, Hampton, VA, May 1988.